Mathematics

Edexcel IAL

S1

Worksheets

The Normal Distribution

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The Normal Distribution

Exercise 1:

1 State, with a reason, whether these random variables are discrete or continuous:

a X, the lengths of a random sample of 100 sidewinder snakes in the Sahara desert

b Y, the scores achieved by 250 students in a university entrance exam

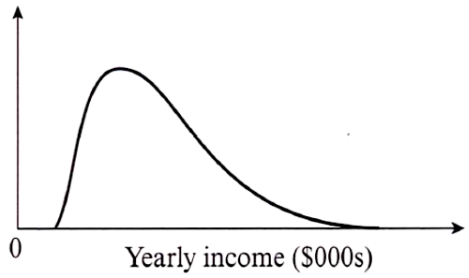
c C, the masses of honey badgers in a random sample of 1000

d Q, the shoe sizes of 200 randomly selected women in a particular town.

2 The lengths, X mm, of a bolt produced by a particular machine are normally distributed with mean 35 mm and standard deviation 0.4 mm. Sketch the distribution of X.

3 The distribution of incomes, in \$000s per year, of employees at a bank is shown on the right.

State, with reasons, why the normal distribution is not a suitable model for these data.



4 The arm spans of a group of Year 5 students, X cm, are modelled as $X \sim N(120, 16)$.

a State the proportion of students who have an arm span between 116 cm and 124 cm.

b State the proportion of students who have an arm span between 112 cm and 128 cm.

5 The lengths of a group of snakes, Y cm, are modelled as $Y \sim N(100, \sigma^2)$. If 68% of the snakes have a length between 93 cm and 107 cm, find σ^2.

6 The weights of a group of mice, D grams, are modelled as $D \sim N(\mu, 25)$. If 97.5% of the mice weigh less than 70 grams, find μ.

Problem-solving

Draw a sketch of the distribution. Use the symmetry of the distribution and the fact that 95% of the data lies within 2 standard deviations of the mean.

7 The masses of the sheep, M kg, on a farm are modelled as $M \sim N(\mu, \sigma^2)$. If 84% of the sheep weigh more than 52 kg and 97.5% of the sheep weigh more than 47.5 kg, find μ and σ^2.

8 The percentage scores, S, of a group of students in a test are modelled as a normal distribution with mean 45 and standard deviation 15. Find:

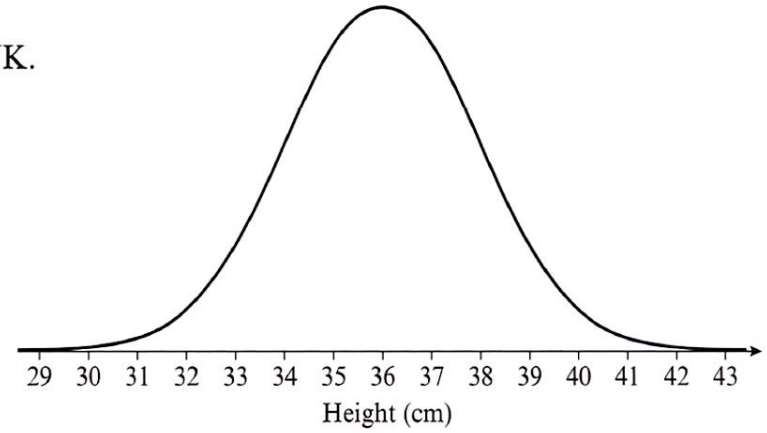
a $P(S > 45)$ **b** $P(30 < S < 60)$ **c** $P(15 < S < 75)$

Alexia states that since it is impossible to score above 100%, this is not a suitable model.

d State, with a reason, whether or not Alexia is correct.

9 The diagram shows the distribution of heights, in cm, of barn owls in the UK.

An ornithologist notices that the distribution is approximately normal.



a State the value of the mean height. **(1 mark)**

b Estimate the standard deviation of the heights. **(2 marks)**

Exercise 2:

1 Use the normal distribution tables to find the following:

a $P(Z > 1.27)$ **b** $P(Z > -1.66)$ **c** $P(Z < -2.28)$

d $P(0 < Z < 1.31)$ **e** $P(1.30 < Z < 1.89)$ **f** $P(-2.8 < Z < -1.6)$

2 For the standard normal distribution $Z \sim N(0, 1^2)$, find:

a $P(Z < 2.12)$ **b** $P(Z < 1.36)$ **c** $P(Z > 0.84)$

d $P(Z < -0.38)$ **e** $P(-2.30 < Z < 0)$ **f** $P(Z < -1.63)$

g $P(-2.16 < Z < -0.85)$ **h** $P(-1.57 < Z < 1.57)$

Exercise 3:

1 Find the value of a in the following:

a $P(Z < a) = 0.3336$ **b** $P(Z > a) = 0.6879$

c $P(Z > a) = 0.1112$ **d** $P(-a < Z < a) = 0.5820$

2 For the standard normal distribution $Z \sim N(0, 1^2)$, find values of a such that:

a $P(Z < a) = 0.9082$ **b** $P(Z > a) = 0.0314$

c $P(Z > a) = 0.1500$ **d** $P(Z > a) = 0.9500$

e $P(0 < Z < a) = 0.3554$ **f** $P(0 < Z < a) = 0.4946$

g $P(-a < Z < a) = 0.80$ **h** $P(-a < Z < a) = 0.40$

Hint For parts **g** and **h** you will need to use the symmetry properties of the distribution.

Exercise 4:

1 The random variable $X \sim N(20, 4^2)$. Find:

a $P(X \leqslant 26)$ **b** $P(X > 30)$ **c** $P(X \geqslant 17)$

2 Given that $X \sim N(18, 10)$, find the following probabilities:

a $P(X > 20)$ **b** $P(X < 15)$

3 The random variable $X \sim N(24, 3^2)$. Find:

a $P(X \leqslant 29)$ **b** $P(X \geqslant 22)$ **c** $P(X < 16)$

4 The random variable $Y \sim N(30, 25)$
Find the value of a such that $P(Y > a) = 0.30$

5 The random variable $Y \sim N(15, 9)$
Find the value of a such that $P(Y > a) = 0.15$

6 The random variable $Y \sim N(100, 225)$
Find the values of s and t such that:

a $P(Y > s) = 0.975$

b $P(Y < t) = 0.10$

c Write down $P(s < Y < t)$

7 Given that $X \sim N(80, 4^2)$,

a find the values a and b when:

i $P(X > a) = 0.40$

ii $P(X < b) = 0.5636$

b Write down $P(b < X < a)$

8 The random variable $X \sim N(0.8, 0.05^2)$. For each of the following values of X, write down the corresponding value of the standardised normal distribution, $Z \sim N(0, 1^2)$.

a $x = 0.8$ **b** $x = 0.792$ **c** $x = 0.81$ **d** $x = 0.837$

9 The normal distribution $X \sim N(154, 12^2)$. Write in terms of $\Phi(z)$:

a $P(X < 154)$ **b** $P(X < 160)$

c $P(X > 151)$ **d** $P(140 < X < 155)$

Hint Write your answer to part **d** in the form $\Phi(z_1) - \Phi(z_2)$

10 **a** Use the percentage points table to find a value of z such that $P(Z > z) = 0.025$ **(1 mark)**

b A pilot training programme takes only the top 2.5% of candidates on a test. Given that the scores can be modelled using a normal distribution with mean 80 and standard deviation 4, use your answer to part **a** to find the score necessary to get on the programme. **(2 marks)**

11 **a** Use the percentage points table to find a value of z such that $P(Z < z) = 0.15$ **(1 mark)**

b A hat manufacturer makes a special 'little' hat which should fit 15% of its customers. Given that hat sizes can be modelled using a normal distribution with mean 57 cm and standard deviation 2 cm, use your answer to part **a** to find the size of a 'little' hat. **(2 marks)**

12 **a** Use the percentage points table to find the values of z that correspond to the 10% to 90% interpercentile range. **(2 marks)**

A particular brand of light bulb has a life modelled as a normal distribution with mean 1175 hours and standard deviation 56 hours. The bulb life is considered 'standard' if its life falls into the 10% to 90% interpercentile range.

b Use your answer to part **a** to find the range of life to the nearest hour for a 'standard' bulb. **(2 marks)**

Exercise 5:

1 The random variable $X \sim N(\mu, 5^2)$ and $P(X < 18) = 0.9032$
Find the value of μ.

2 The random variable $X \sim N(11, \sigma^2)$ and $P(X > 20) = 0.01$
Find the value of σ.

3 The random variable $Y \sim N(\mu, 40)$ and $P(Y < 25) = 0.15$
Find the value of μ.

4 The random variable $Y \sim N(50, \sigma^2)$ and $P(Y > 40) = 0.6554$
Find the value of σ.

5 The random variable $X \sim N(\mu, \sigma^2)$.
Given that $P(X < 17) = 0.8159$ and $P(X < 25) = 0.9970$, find the value of μ and the value of σ.

6 The random variable $Y \sim N(\mu, \sigma^2)$.
Given that $P(Y < 25) = 0.10$ and $P(Y > 35) = 0.005$, find the value of μ and the value of σ.

7 The random variable $X \sim N(\mu, \sigma^2)$.
Given that $P(X > 15) = 0.20$ and $P(X < 9) = 0.20$,
find the value of μ and the value of σ.

Hint Draw a diagram and use symmetry to find μ.

8 The random variable $X \sim N(\mu, \sigma^2)$.
The lower quartile of X is 25 and the upper quartile of X is 45.
Find the value of μ and the value of σ.

9 The random variable $X \sim N(0, \sigma^2)$.
Given that $P(-4 < X < 4) = 0.6$, find the value of σ.

10 The random variable $X \sim N(2.68, \sigma^2)$.
Given that $P(X > 2a) = 0.2$ and $P(X < a) = 0.4$, find the value of σ and the value of a.

11 An automated wheel is used to make bowls. The diameter of the bowls, D mm, is normally distributed with mean μ and standard deviation 5 mm. Given that 75% of bowls are greater than 200 mm in diameter, find:

a the value of μ **(2 marks)**

b $P(204 < D < 206)$ **(1 mark)**

Three bowls are chosen at random.

c Find the probability that all three bowls are greater than 205 mm in diameter. **(3 marks)**

12 A loom makes tablecloths with an average thickness of 2.5 mm. The thickness, T mm, can be modelled using a normal distribution. Given that 65% of tablecloths are less than 2.55 mm thick, find:

a the standard deviation of the thickness **(2 marks)**

b the proportion of tablecloths with thickness between 2.4 mm and 2.6 mm. **(1 mark)**

A tablecloth can be sold if the thickness is between 2.4 mm and 2.6 mm.
A sample of 20 tablecloths is taken.

c Find the probability that at least 15 tablecloths can be sold. **(3 marks)**

13 The masses of the penguins on an island are found to be normally distributed with mean μ, and standard deviation σ. Given that 10% of the penguins have a mass less than 18 kg and 5% of the penguins have a mass greater than 30 kg,

a sketch a diagram to represent this information **(2 marks)**

b find the value of μ and the value of σ. **(6 marks)**

10 penguins are chosen at random.

c Find the probability that at least 4 of them have a mass greater than 25 kg. **(4 marks)**

14 The length of an adult Dachshund dog is found to be normally distributed with mean μ and standard deviation σ. Given that 20% of Dachshunds have a length less than 16 inches and 10% have a length greater than 18 inches, find:

a the value of μ and the value of σ **(6 marks)**

b the interquartile range. **(2 marks)**