British Maths

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number	Candidate Number

Pearson Edexcel International Advanced Level

Thursday 9 May 2024

Morning (Time: 1 hour 30 minutes)

Paper reference **WMA11/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 11 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

Pearson

1. Find

$$\int\left(10x^4 - \frac{3}{2x^3} - 7\right)\mathrm{d}x$$

giving each term in simplest form.

(3)

$$\int\left(10x^4 - \frac{3}{2}x^{-3} - 7\right)dx$$

$$= \frac{10x^5}{5} - \frac{3}{2}\,\frac{x^{-2}}{-2} - 7x + C$$

$$= 2x^5 + \frac{3}{4}x^{-2} - 7x + C$$

2. (i) Given that $m = 2^n$, express each of the following in simplest form in terms of m.

(a) 2^{n+3}

(1)

(b) 16^{3n}

(2)

(ii) **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Solve the equation

$$x\sqrt{3} - 3 = x + \sqrt{3}$$

giving your answer in the form $p + q\sqrt{3}$ where p and q are integers.

(3)

(i)

(a) $2^{n+3} = 2^n \times 2^3 = 8m$

(b) $16^{3n} = (2^4)^{3n} = 2^{12n} = m^{12}$

(ii) $x\sqrt{3} - x = 3 + \sqrt{3}$

$x(\sqrt{3} - 1) = 3 + \sqrt{3}$

$x = \dfrac{(3+\sqrt{3})(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \dfrac{3\sqrt{3} + 3 + 3 + \sqrt{3}}{3 - 1}$

$= \dfrac{6 + 4\sqrt{3}}{2} = 3 + 2\sqrt{3}$

3.

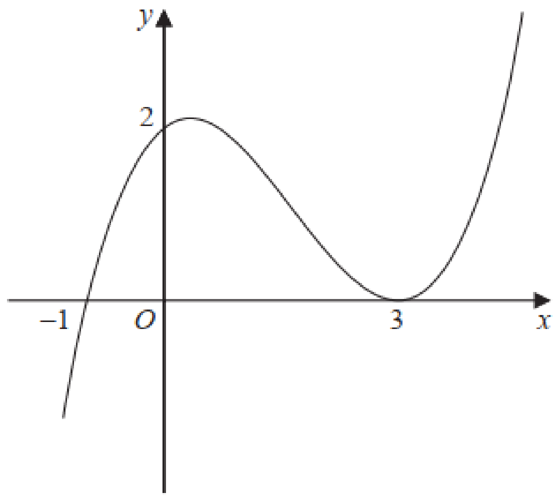


Figure 1

Figure 1 shows a sketch of the curve with equation $y = f(x)$.

The curve passes through the points $(-1, 0)$ and $(0, 2)$ and touches the x-axis at the point $(3, 0)$.

On separate diagrams, sketch the curve with equation

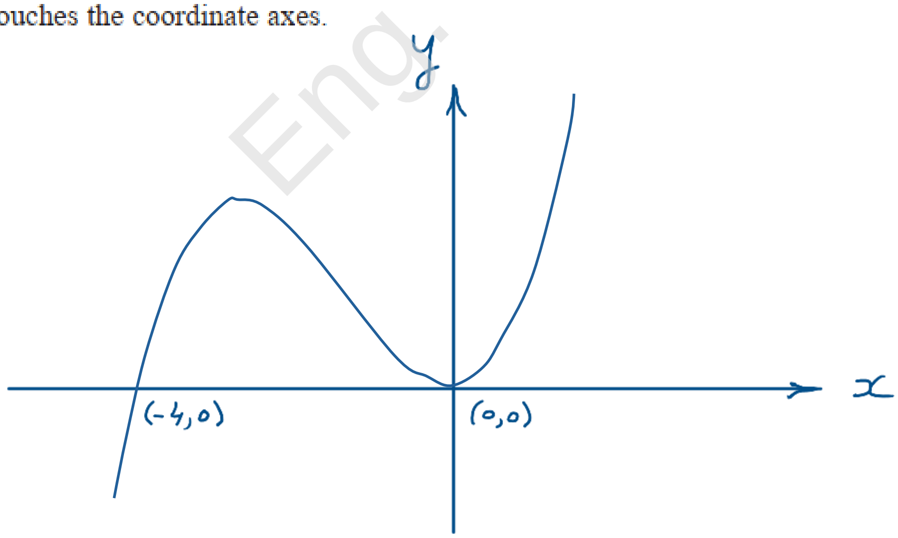
(a) $y = f(x + 3)$

(3)

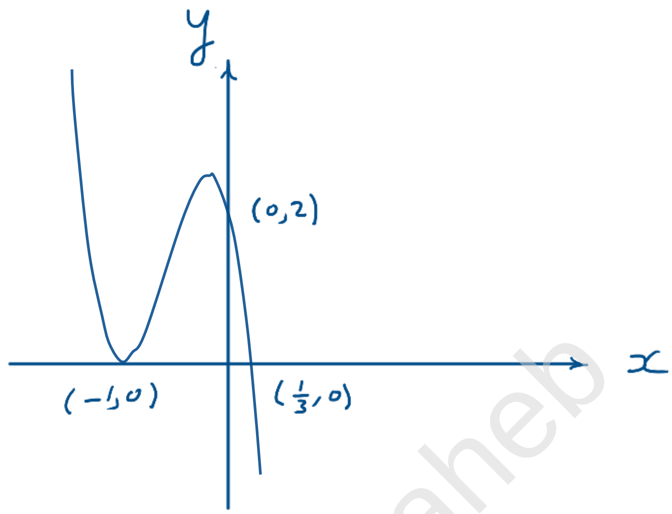
(b) $y = f(-3x)$

(3)

On each diagram, show clearly the coordinates of all the points where the curve cuts or touches the coordinate axes.



Question 3 continued



(Total for Question 3 is 6 marks)

4. The curve C_1 has equation

$$y = x^2 + kx - 9$$

and the curve C_2 has equation

$$y = -3x^2 - 5x + k$$

where k is a constant.

Given that C_1 and C_2 meet at a single point P

(a) show that

$$k^2 + 26k + 169 = 0$$

(3)

(b) Hence find the coordinates of P

(3)

British Maths

(a) $x^2 + kx - 9 = -3x^2 - 5x + k$

$4x^2 + (k+5)x - 9 - k = 0$

$b^2 - 4ac = 0 \qquad (k+5)^2 - 4(4)(-9-k) = 0$

$k^2 + 10k + 25 - 16(-9-k) = 0$

$k^2 + 10k + 25 + 144 + 16k = 0$

$k^2 + 26k + 169 = 0$

(b) $(k+13)(k+13) = 0$

$k = -13$

$4x^2 - 8x + 4 = 0 \quad \div 4$

$x^2 - 2x + 1 = 0$

$(x-1)(x-1) = 0$

$x = 1$

$y = 1 - 13(1) - 9 = -21 \qquad P(1, -21)$

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5.

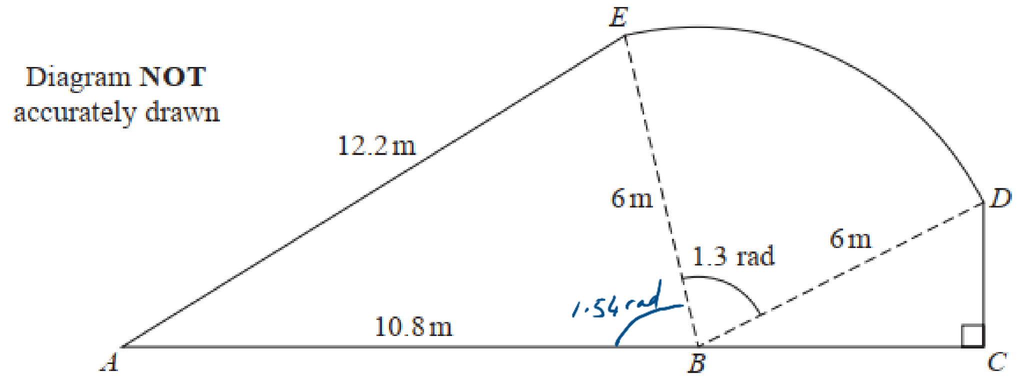


Figure 2

Figure 2 shows the plan view of a garden.

The shape of the garden $ABCDEA$ consists of a triangle ABE and a right-angled triangle BCD joined to a sector BDE of a circle with radius 6 m and centre B.

The points A, B and C lie on a straight line with $AB = 10.8$ m

Angle $BCD = \frac{\pi}{2}$ radians, angle $EBD = 1.3$ radians and $AE = 12.2$ m

(a) Find the area of the sector BDE, giving your answer in m^2 **(2)**

(b) Find the size of angle ABE, giving your answer in radians to 2 decimal places. **(2)**

(c) Find the area of the garden, giving your answer in m^2 to 3 significant figures. **(3)**

(a) Area of sector $= \frac{1}{2} r^2 \theta^{rad} = \frac{1}{2} \times 6^2 \times 1.3 = 23.4 \ m^2$

(b) $\angle ABE = \cos^{-1}\left(\frac{6^2 + 10.8^2 - 12.2^2}{2 \times 6 \times 10.8} \right)$

$\angle ABE = 1.54$ rad

(c) Area of $\triangle ABE = \frac{1}{2} \times 6 \times 10.8 \sin 1.54 = 32.38 \ m^2$

$\angle CBD = \pi - (1.54 + 1.3) = 0.30159$ rad

$BC = 6 \cos 0.30159 = 5.729$ m

Area of $\triangle BCD = \frac{1}{2} \times 6 \times 5.729 \sin 0.30159 = 5.105 \ m^2$

Question 5 continued

Area $= 23.4 + 32.38 + 5.105 = 60.9 \, m^2$

6.

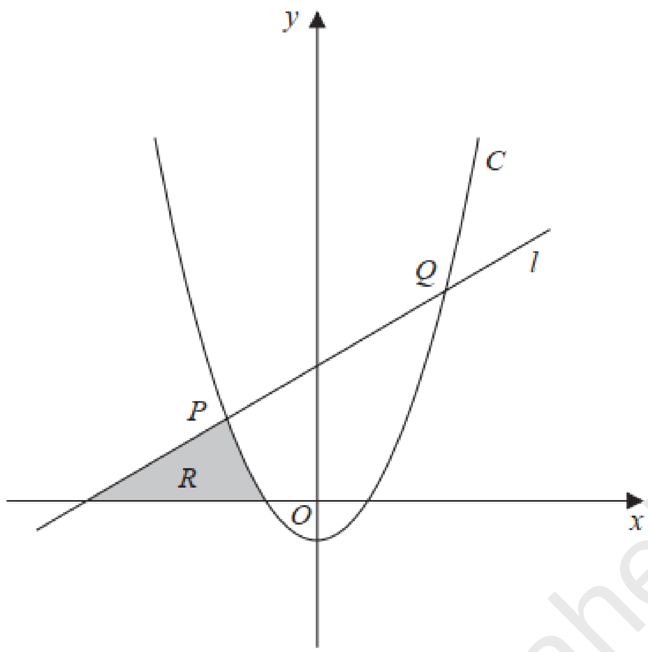


Figure 3

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Figure 3 shows

- the line l with equation $y - 5x = 75$
- the curve C with equation $y = 2x^2 + x - 21$

The line l intersects the curve C at the points P and Q, as shown in Figure 3.

(a) Find, using algebra, the coordinates of P and the coordinates of Q.

(4)

The region R, shown shaded in Figure 3, is bounded by C, l and the x-axis.

(b) Use inequalities to define the region R.

(3)

(a) $2x^2+x-21=75+5x$

$2x^2-4x-96=0 \quad \div 2$

$x^2-2x-48=0$

$(x-8)(x+6)=0 \qquad x=8 \,,\, x=-6$

Question 6 continued

at $x=-6$, $y-5(-6)=75$ $y=45$

at $x=8$, $y=-5(8)=75$ $y=115$

$P(-6,45)$, $Q(8,115)$

(b) $y \geqslant 0$, $y \leqslant 5x+75$, $y \leqslant 2x^2+x-21$, $x \leqslant 0$

$-3.5 \leqslant a \leqslant 3$

7. The curve C has equation $y = f(x)$ where

$$f(x) = 2x^3 - kx^2 + 14x + 24$$

and k is a constant.

(a) Find, in simplest form,

(i) $f'(x)$

(ii) $f''(x)$

(3)

The curve with equation $y = f'(x)$ intersects the curve with equation $y = f''(x)$ at the points A and B.

Given that the x coordinate of A is 5

(b) find the value of k.

(2)

(c) Hence find the coordinates of B.

(3)

(a) (i) $f'(x) = 6x^2 - 2kx + 14$

(ii) $f''(x) = 12x - 2k$

(b) $6x^2 - 2kx + 14 = 12x - 2k$

$6x^2 - 2kx - 12x + 14 + 2k = 0 \quad \div 2$

$3x^2 - kx - 6x + 7 + k = 0$

at $x = 5$ $\quad 3(5)^2 - k(5) - 6(5) + 7 + k = 0$

$75 - 5k - 23 + k = 0$

$\cancel{-}4k = \cancel{-}52 \qquad k = 13$

(c) $3x^2 - 13x - 6x + 7 + 13 = 0$

$3x^2 - 19x + 20 = 0$

Question 7 continued

$(x-5)(3x-4)=0$

at B $\quad x=\frac{4}{3} \qquad y=6\left(\frac{4}{3}\right)^2-2\times 13\left(\frac{4}{3}\right)+14=-10$

$B\left(\frac{4}{3},-10\right)$

(Total for Question 7 is 8 marks)

8. The curve C_1 has equation

$$y = x(4 - x^2)$$

(a) Sketch the graph of C_1 showing the coordinates of any points of intersection with the coordinate axes.

(3)

The curve C_2 has equation $y = \frac{A}{x}$ where A is a constant.

(b) Show that the x coordinates of the points of intersection of C_1 and C_2 satisfy the equation

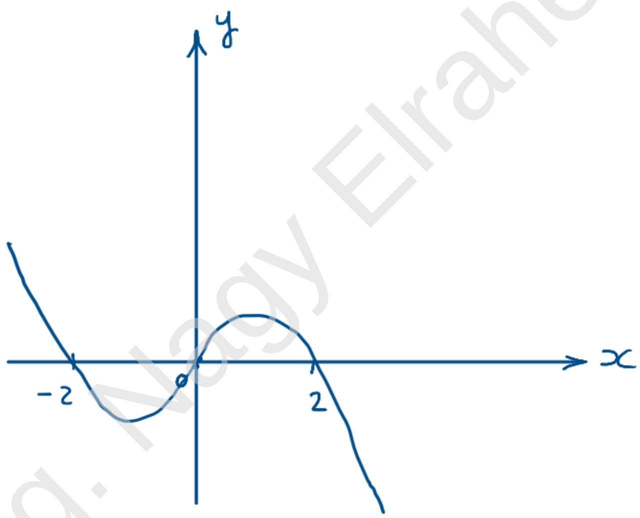
$$x^4 - 4x^2 + A = 0$$

(1)

(c) Hence find the range of possible values of A for which C_1 meets C_2 at 4 distinct points.

(3)

(a)



(b) $x(4 - x^2) = \frac{A}{x}$

$x^2(4 - x^2) = A$

$4x^2 - x^4 = A$ $\qquad x^4 - 4x^2 + A = 0$

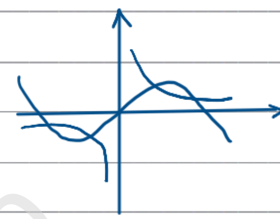
Question 8 continued

(c) $b^2 - 4ac > 0$

$16 - 4(1)A > 0$ $\qquad$ $16 - 4A > 0$

$4A < 16$ $\qquad$ $A < 4$

For an intersection of 4 points



$\frac{A}{x}$ has to be a +ve A $\qquad$ $A > 0$

$0 < A < 4$

(Total for Question 8 is 7 marks)

9. Given that

- the point A has coordinates (4, 2)
- the point B has coordinates (15, 7)
- the line l_1 passes through A and B

(a) find an equation for l_1, giving your answer in the form $px + qy + r = 0$ where p, q and r are integers to be found.

(3)

The line l_2 passes through A and is parallel to the x-axis.

The point C lies on l_2 so that the length of BC is $5\sqrt{5}$

(b) Find both possible pairs of coordinates of the point C.

(4)

(c) Hence find the minimum possible area of triangle ABC.

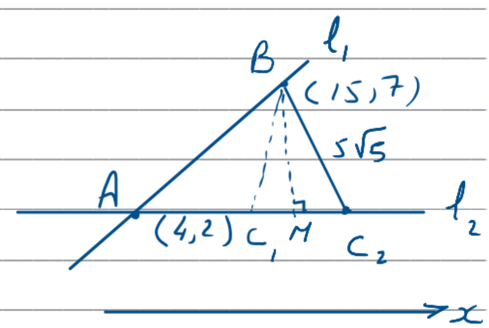
(2)

(a) $m = \dfrac{7-2}{15-4} = \dfrac{5}{11}$

$y - 2 = \dfrac{5}{11}(x-4)$

$11y - 22 = 5x - 20$

$5x - 11y + 2 = 0$



(b) $l_2: \; y = 2$

in Δ BMC_2 $\quad BM = 7 - 2 = 5$

$MC_2 = \sqrt{(5\sqrt{5})^2 - 5^2} = 10$

$C_1 = 15 - 10 = 5$

$C_2 = 15 + 10 = 25$

So $C(5,2)$ or $C(25,2)$

(c) Min area $= \frac{1}{2} \times (5-4) \times 5 = 5/2$

10. The curve C has equation $y = f(x)$ where $x > 0$

Given that

- $f'(x) = 6x - \dfrac{(2x-1)(3x+2)}{2\sqrt{x}}$
- the point $P(4, 12)$ lies on C

(a) find the equation of the normal to C at P, giving your answer in the form $y = mx + c$ where m and c are integers to be found,

(4)

(b) find $f(x)$, giving each term in simplest form.

(6)

(a) At P $\quad f'(x) = 6(4) - \dfrac{(8-1)(12+2)}{2\sqrt{4}} = -\dfrac{1}{2}$

Gradient of the normal $= 2$

Eqn. of the normal : $y - 12 = 2(x-4)$

$y - 12 = 2x - 8 \qquad y = 2x + 4$

(b) $f(x) = \displaystyle\int \left(6x - \frac{6x^2 + 4x - 3x - 2}{2\sqrt{x}}\right) dx$

$= \displaystyle\int \left(6x - \frac{6x^2}{2x^{1/2}} - \frac{x}{2x^{1/2}} + \frac{2}{2x^{1/2}}\right) dx$

$= \displaystyle\int \left(6x - 3x^{3/2} - \frac{1}{2}x^{1/2} + x^{-1/2}\right) dx$

$= \dfrac{6x^2}{2} - 3x^{5/2} \times \dfrac{2}{5} - \dfrac{1}{2}x^{3/2} \times \dfrac{2}{3} + x^{1/2} \times 2 + C$

$= 3x^2 - \dfrac{6}{5}x^{5/2} - \dfrac{1}{3}x^{3/2} + 2x^{1/2} + C$

$12 = 3(4)^2 - \dfrac{6}{5}(4)^{5/2} - \dfrac{1}{3}(4)^{3/2} + 2(4)^{1/2} + C$

$12 = \dfrac{164}{15} + C \qquad C = \dfrac{16}{15}$

$f(x) = 3x^2 - \dfrac{6}{5}x^{5/2} - \dfrac{1}{3}x^{3/2} + 2x^{1/2} + \dfrac{16}{15}$

11.

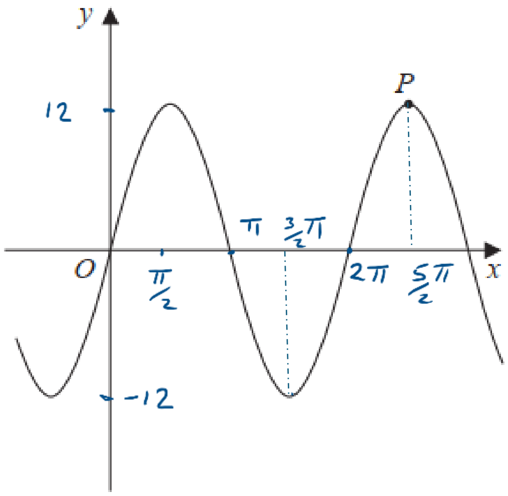


Figure 4

Figure 4 shows a sketch of part of the curve C_1 with equation

$$y = 12\sin x$$

where x is measured in radians.

The point P shown in Figure 4 is a maximum point on C_1

(a) Find the coordinates of P.

(2)

The curve C_2 has equation

$$y = 12\sin x + k$$

where k is a constant.

Given that the **maximum** value of y on C_2 is 3

(b) find the coordinates of the **minimum** point on C_2 which has the **smallest** positive x coordinate.

(2)

The curve C_3 has equation

$$y = 12\sin(x + B)$$

where B is a positive constant.

Given that $\left(\frac{\pi}{4}, A\right)$, where A is a constant, is the **minimum** point on C_3 which has the **smallest** positive x coordinate,

(c) find

(i) the value of A,

(ii) the smallest possible value of B.

(2)

Question 11 continued

(a) $P\left(\frac{5}{2}\pi, 12\right)$

(b) $12 - 3 = 9$

Minimum point $\left(\frac{3}{2}\pi, -21\right)$

(c) (i) $A = -12$

(ii) $B = \frac{3}{2}\pi - \frac{\pi}{4} = \frac{5}{4}\pi$

Question 11 continued

(Total for Question 11 is 6 marks)

TOTAL FOR PAPER IS 75 MARKS