Mathematics

Edexcel IAL

S1

Worksheets

Discrete Random Variables

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Discrete Random Variables

Exercise 1:

1 Write down whether or not each of the following is a discrete random variable.
Give a reason for each answer.

a The height, X cm, of a seedling chosen randomly from a group of plants

b The number of times, R, a six appears when a fair dice is rolled 100 times

c The number of days, W, in a given week

2 A fair dice is rolled four times and the number of times it falls with a 6 on the top, Y, is noted.
Write down the sample space of Y.

3 A bag contains two discs with the number 2 on them and two discs with the number 3 on them.
A disc is drawn at random from the bag and the number noted. The disc is returned to the bag.
A second disc is then drawn from the bag and the number noted.

a Write down all the possible outcomes of this experiment.

The discrete random variable X is defined as the sum of the two numbers.

b Write down the probability distribution of X as:
i a table **ii** a probability function.

4 A discrete random variable X has the probability distribution shown in the table.
Find the value of k.

x	1	2	3	4
$P(X = x)$	$\frac{1}{3}$	$\frac{1}{3}$	k	$\frac{1}{4}$

5 The random variable X has a probability function

$$P(X = x) = kx \qquad x = 1, 2, 3, 4$$

Show that $k = \frac{1}{10}$ **(2 marks)**

6 The random variable X has a probability function

$$P(X = x) = \begin{cases} kx & x = 1, 3 \\ k(x - 1) & x = 2, 4 \end{cases}$$

where k is a constant.

a Find the value of k. **(2 marks)**

b Find $P(X > 1)$. **(2 marks)**

7 The discrete random variable X has a probability function

$$P(X = x) = \begin{cases} 0.1 & x = -2, -1 \\ \beta & x = 0, 1 \\ 0.2 & x = 2 \end{cases}$$

a Find the value of β.

b Construct a table giving the probability distribution of X.

c Find $P(-1 \leqslant X < 2)$.

8 A discrete random variable has a probability distribution shown in the table.

Find the value of a.

x	0	1	2
$P(X = x)$	$\frac{1}{4} - a$	a	$\frac{1}{2} + a$

9 The random variable X can take any integer value from 1 to 50. Given that X has a discrete uniform distribution, find:

a $P(X = 1)$

b $P(X \geqslant 28)$

c $P(13 < X < 42)$

10 A discrete random variable X has the probability distribution shown in this table.

x	0	1	2	3
$P(X = x)$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{8}$

Find:

a $P(1 < X \leqslant 3)$ **(1 mark)**

b $P(X < 2)$ **(1 mark)**

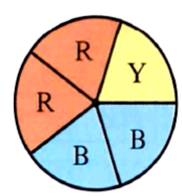
c $P(X > 3)$ **(1 mark)**

11 A biased coin is tossed until a head appears or it is tossed four times.

a If $P(\text{Head}) = \frac{2}{3}$, write down the probability distribution of S, the number of tosses, in table form. **(4 marks)**

b Find $P(S > 2)$. **(1 mark)**

12 A fair five-sided spinner is spun.



Given that the spinner is spun five times, write down, in table form, the probability distributions of the following random variables:

a X, the number of times red appears

b Y, the number of times yellow appears.

The spinner is now spun until it lands on blue, or until it has been spun five times. The random variable Z is defined as the number of spins in this experiment.

c Find the probability distribution of Z.

13 Marie says that a random variable X has a probability distribution defined by the following probability function:

$$P(X = x) = \frac{2}{x^2} \quad x = 2, 3, 4$$

a Explain how you know that Marie's function does not describe a probability distribution. **(2 marks)**

b Given that the correct probability function is in the form:

$$P(X = x) = \frac{k}{x^2}, \quad x = 2, 3, 4$$

where k is a constant, find the exact value of k. **(2 marks)**

Exercise 2:

1 A discrete random variable X has a probability distribution given in the table.

x	1	2	3	4	5	6
$P(X = x)$	0.1	0.1	0.15	0.25	0.3	0.1

a Draw a table showing the cumulative distribution function $F(x)$.

b Write down $F(5)$

c Write down $F(2.2)$

2 A discrete random variable has a cumulative distribution function $F(x)$ given in the table

x	0	1	2	3	4	5	6
$F(x)$	0	0.1	0.2	0.45	0.5	0.9	1

a Show, by drawing a table, the probability distribution of X.

b Write down $P(X < 5)$

c Find $P(2 \leqslant X < 5)$

3 The random variable X has a probability function

$$P(X = x) = \begin{cases} kx & x = 1, 3, 5 \\ k(x-1) & x = 2, 4, 6 \end{cases}$$

where k is a constant.

a Find the value of k.

b Draw a table giving the probability distribution of X.

c Find $P(2 \leqslant X < 5)$

d Find $F(4)$

e Find $F(1.6)$

4 The discrete random variable X has a probability function

$$P(X = x) = \begin{cases} 0.1 & x = -2, -1 \\ \alpha & x = 0, 1 \\ 0.3 & x = 2 \end{cases}$$

a Find the value of α.

b Draw a table giving the probability distribution of X.

c Write down the value of F(0.3)

5 The discrete random variable X has a probability function P(x) defined by

$$P(X = x) = \begin{cases} 0 & x = 0 \\ \dfrac{1 + x}{6} & x = 1, 2, 3, 4, 5 \\ 1 & x > 5 \end{cases}$$

a Find $P(X \leqslant 4)$

b Show that $P(X = 4)$ is $\frac{1}{6}$

c Find the probability distribution for X.

6 The discrete random variable X has a cumulative distribution function F(x) defined by

$$F(x) = \begin{cases} 0 & x = 0 \\ \dfrac{(x + k)^2}{16} & x = 1, 2 \text{ and } 3 \\ 1 & x > 5 \end{cases}$$

a Find the value of k.

b Find the probability distribution for X.

Exercise 3:

1 For each of the following probability distributions, write out the distribution of X^2 and calculate both $E(X)$ and $E(X^2)$.

a

x	2	4	6	8
$P(X = x)$	0.3	0.3	0.2	0.2

b

x	−2	−1	1	2
$P(X = x)$	0.1	0.4	0.1	0.4

2 The score on a biased dice is modelled by a random variable X with probability distribution:

x	1	2	3	4	5	6
P($X = x$)	0.1	0.1	0.1	0.2	0.4	0.1

Find E(X) and E(X^2).

3 The random variable X has a probability function:

$$P(X = x) = \frac{1}{x}; \qquad x = 2, 3, 6$$

a Construct tables giving the probability distributions of X and X^2.

b Work out E(X) and E(X^2).

c State whether or not $(E(X))^2 = E(X^2)$.

4 The random variable X has a probability function given by

$$P(X = x) = \begin{cases} 2^{-x} & x = 1, 2, 3, 4 \\ 2^{-4} & x = 5 \end{cases}$$

a Construct a table giving the probability distribution of X.

b Calculate E(X) and E(X^2).

c State whether or not $(E(X))^2 = E(X^2)$.

5 The random variable X has the following probability distribution:

x	1	2	3	4	5
P($X = x$)	0.1	a	b	0.2	0.1

Given that E(X) = 2.9, find the value of a and the value of b. **(5 marks)**

6 The discrete random variable X has probability function:

$$P(X = x) = \begin{cases} a(1 - x) & x = -2, -1, 0 \\ b & x = 5 \end{cases}$$

Given that E(X) = 1.2, find the value of a and the value of b.

7 A biased six-sided dice has a $\frac{1}{8}$ chance of landing on any of the numbers 1, 2, 3 or 4. The probabilities of landing on 5 or 6 are unknown. The outcome is modelled as a random variable, X. Given that E(X) = 4.1, find the probability distribution of X. **(5 marks)**

8 A company makes phone covers. One out of every 50 phone covers is faulty, but the company doesn't know which ones are faulty until a buyer complains. Suppose the company makes a \$3 profit on the sale of any working phone cover, but suffers a loss of \$8 for every faulty phone cover due to replacement costs. Calculate the expected profit for each phone cover, regardless of whether or not it is faulty. **(5 marks)**

Exercise 4:

1 The random variable X has a probability distribution given by:

x	-1	0	1	2	3
$P(X = x)$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

a Find $E(X)$

b Find $Var(X)$

2 Find the expected value and variance of the random variable X with probability distributions given by the following tables:

a

x	1	2	3
$P(X = x)$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$

b

x	-1	0	1
$P(X = x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

c

x	-2	-1	1	2
$P(X = x)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$

3 Given that Y is the score when a single, unbiased, eight-sided dice is rolled, find $E(Y)$ and $Var(Y)$.

4 Two fair, cubical (six-sided) dice are rolled and S is the sum of their scores. Find:

a the distribution of S

b $E(S)$

c $Var(S)$

d the standard deviation, σ

Hint The standard deviation of a random variable is the square root of its variance.

5 Two fair, tetrahedral (four-sided) dice are rolled and D is the difference between their scores. Find:

a the distribution of D

b $E(D)$

c $Var(D)$

6 A fair coin is flipped repeatedly until a head appears or three flips have been made. The random variable T represents the number of flips of the coin.

a Show that the probability distribution of T is:

t	1	2	3
P($T = t$)	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

(3 marks)

b Find the expected value and variance of T. **(6 marks)**

7 The random variable X has a probability distribution given by:

x	1	2	3
P($X = x$)	a	b	a

where a and b are constants.

a Write down an expression for $E(X)$ in terms of a and b. **(2 marks)**

b Given that $\text{Var}(X) = 0.75$, find the values of a and b. **(5 marks)**

Exercise 5:

1 The random variable X has a probability distribution given by:

x	1	2	3	4
P($X = x$)	0.1	0.3	0.2	0.4

a Write down the probability distribution for Y where $Y = 2X - 3$

b Find $E(Y)$

c Calculate $E(X)$ and verify that $E(2X - 3) = 2E(X) - 3$

2 The random variable X has a probability distribution given by:

x	−2	−1	0	1	2
P($X = x$)	0.1	0.1	0.2	0.4	0.2

a Write down the probability distribution for Y where $Y = X^3$

b Calculate $E(Y)$

3 The random variable X has $E(X) = 1$ and $Var(X) = 2$. Find:

a $E(8X)$ **b** $E(X + 3)$ **c** $Var(X + 3)$

d $Var(3X)$ **e** $Var(1 - 2X)$ **f** $E(X^2)$

4 The random variable X has $E(X) = 3$ and $E(X^2) = 10$. Find:

a $E(2X)$ **b** $E(3 - 4X)$ **c** $E(X^2 - 4X)$

d $Var(X)$ **e** $Var(3X + 2)$

5 The random variable X has a mean μ and standard deviation σ.

Find, in terms of μ and σ:

a $E(4X)$ **b** $E(2X + 2)$ **c** $E(2X - 2)$

d $Var(2X + 2)$ **e** $Var(2X - 2)$

6 In a space-themed board game, players roll a fair, six sided dice each time they make it around the board. The board represents one turn around the galaxy. The score on the dice is modelled as a discrete random variable X.

a Write down $E(X)$.

Players collect 200 points, plus 100 times the score on the dice. The amount of points given to each player is modelled as a discrete random variable Y.

b Write Y in terms of X.

c Find the expected number of points a player receives each time they make it around the board.

7 Hiroki runs a pizza parlour that sells pizza in three sizes: small (20 cm diameter), medium (30 cm diameter) and large (40 cm diameter). Each pizza base is 1 cm thick. Hiroki has worked out that on average, customers order a small, medium or large pizza with probabilities $\frac{3}{10}$, $\frac{9}{20}$ and $\frac{5}{20}$ respectively. Calculate the expected amount of pizza dough needed per customer.

8 Two tetrahedral dice are rolled. The random variable X represents the result of subtracting the smaller score from the larger.

a Find $E(X)$ and $Var(X)$. **(7 marks)**

The random variables Y and Z are defined as $Y = 2^X$ and $Z = \frac{4X + 1}{2}$

b Show that $E(Y) = E(Z)$. **(3 marks)**

c Find $Var(Z)$. **(2 marks)**

Exercise 6:

1 X is a discrete random variable. The random variable Y is defined by $Y = 4X - 6$
Given that $E(Y) = 2$ and $Var(Y) = 32$, find:

a $E(X)$

b $Var(X)$

c the standard deviation of X.

2 X is a discrete random variable. The random variable Y is defined by $Y = \frac{4 - 3X}{2}$

Given that $E(Y) = -1$ and $Var(Y) = 9$, find:

a $E(X)$

b $Var(X)$

c $E(X^2)$

3 The discrete random variable X has a probability distribution given by:

x	1	2	3	4
$P(X = x)$	0.3	a	b	0.2

The random variable Y is defined by $Y = 2X + 3$. Given that $E(Y) = 8$, find the values of a and b.

4 The discrete random variable X has a probability distribution given by:

x	90°	180°	270°
$P(X = x)$	a	b	0.3

The random variable Y is defined as $Y = \sin X°$

a Find the range of possible values of $E(Y)$. **(5 marks)**

b Given that $E(Y) = 0.2$, write down the values of a and b. **(2 marks)**

Exercise 7:

1 X is a discrete uniform distribution over the numbers 1, 2, 3, 4 and 5.
Work out the expectation and variance of X.

2 Seven similar balls are placed in a bag. The balls have the numbers 1 to 7 written on them. A ball is drawn out of the bag at random. The variable X represents the number on the ball.

a Find $E(X)$

b Work out $Var(X)$

3 A fair dice is thrown once and the random variable X represents the value on the upper face.

a Find the expectation and variance of X.

b Calculate the probability that X is within one standard deviation of the mean.

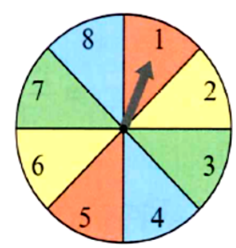
4 A card is selected at random from a pack of cards containing the even numbers 2, 4, 6 …, 20. The variable X represents the number on the card.

a Find $P(X > 15)$

b Find the expectation and variance of X.

5 A straight line is drawn on a piece of paper. The line is divided into four equal lengths and the segments are marked 1, 2, 3 and 4. In a party game, a person is blindfolded and asked to mark a point on the line and then the number of the segment is recorded. A discrete uniform distribution over the set $(1, 2, 3, 4)$ is suggested as a model for this distribution. Comment on this suggestion.

6 The spinner shown is used in a fairground game. It costs 5 cents to have a go on the spinner.



The spinner is spun and the player wins the number of cents shown.

If X is the number which comes up on the next spin,

a name a suitable model for X

b find $E(X)$

c find $Var(X)$

d explain why a player should not expect to make money over a large number of spins.