



Mark Scheme (Results)

January 2024

Pearson Edexcel International Advanced Level
in Decision Mathematics D1 (WDM11) Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN:

- bod – benefit of doubt
 - ft – follow through
 - the symbol \surd will be used for correct ft
 - cao – correct answer only
 - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC – special case
 - oe – or equivalent (and appropriate)
 - d... or dep – dependent
 - indep – independent
 - dp – decimal places
 - sf – significant figures
 - * – The answer is printed on the paper or ag- answer given
 - \square or d... – The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread

however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.
6. If a candidate makes more than one attempt at any question:
 - a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
7. Ignore wrong working or incorrect statements following a correct answer.

Question Number	Scheme	Marks
1. (a)		M1 A1 M1 A1 (4)
(b)	Activity D has a total float of $12 - 4 - 5 = 3$ (hours)	B1ft (1)
(c)	Lower bound = $\frac{4 + 3 + 7 + \dots + 5 + 12}{24} = \frac{67}{24} = 2.791\dots = 3$ workers	M1 A1 (2)
(d)		M1 A1 A1 A1 (4)
(e)	Minimum is 4 workers e.g. activities F, H, I and L together with $12 < \text{time} < 13$	dM1 A1 (2)
13 marks		
Notes for Question 1		
a1M1	All top boxes complete, values generally increasing in the direction of the arrows ('left to right'), condone one rogue	
a1A1	CAO (top boxes)	
a2M1	All bottom boxes complete, values generally decreasing in the opposite direction of the arrows ('right to left'), condone one rogue. Condone missing 0 and/or their 24 (at the end event) for the M mark only	
a2A1	CAO (bottom boxes)	
b1B1ft	Correct calculation for their event times for activity D (all three figures must be seen)	

Question Number	Scheme	Marks
c1M1	Attempt to find lower bound: (a value in the interval [55 – 79] / their finish time) or (sum of the activities (12 values) [condone one missing value] / their finish time) or (as a minimum) an awrt 2.8	
c1A1	CSO – requires both a correct calculation or awrt 2.8 seen and 3. An answer of 3 with no working scores no marks	
d1M1	At least eight different activities labelled including at least five floats. A scheduling diagram (so a diagram in which no floats are evident) scores M0	
d1A1	The critical activities dealt with correctly and appearing just once (C, F, G and J) and three non-critical activities dealt with correctly (both duration and total float correct)	
d2A1	Any five non-critical activities correct (this mark is not dependent on the previous A mark)	
d3A1	CSO – completely correct Gantt chart (exactly twelve activities appearing just once)	
e1depM1	Dependent on M mark in (d). Either a statement with the correct number of workers (4) and stating the correct activities (F, H, I and L) with any numerical time stated or the correct number of workers (4) and a time in the interval $12 \leq t \leq 13$ – mark the numerical value only not their use of the words ‘day/time’ (or equivalent)	
e1A1	A completely correct statement with details of both time and activities. Candidates must give a time within the correct interval of $12 < t < 13$, e.g. 12.5 (or ‘on/during hour 13’) and state the correct activities (F, H, I and L). Project is measured in hours, so a time of (e.g.) 12:30 is acceptable. (Condone use of ‘days’ instead of ‘hours’) Please note the strict inequalities for the time interval (e.g. implying a time of 12 is incorrect). Answers given as an interval of time are acceptable provided the time interval stated is correct for all its possible values (e.g. time 12 – 13 or ‘between 12 and 13’ is A0). A completely correct statement with an additional incorrect statement scores A0 (so do not ignore subsequent working)	

For (d) the following may be useful in checking their cascade chart provided the float is shown after the corresponding activity:

Activity	Duration + Float
A	0 to 4 F: 4 to 7
B	0 to 3 F: 3 to 13
C	0 to 7 Critical
D	4 to 9 F: 9 to 12
E	4 to 6 F: 6 to 18

Activity	Duration + Float
F	7 to 13 Critical
G	13 to 18 Critical
H	7 to 13 F: 13 to 18
I	7 to 13 F: 13 to 18
J	18 to 24 Critical

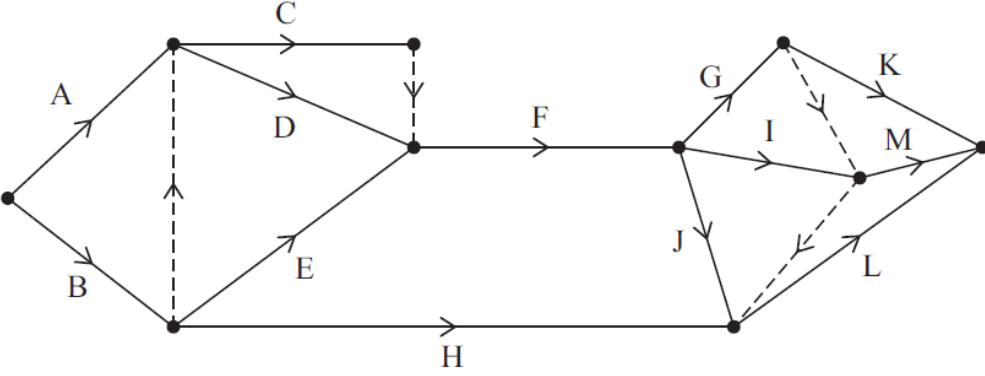
Activity	Duration + Float
K	13 to 18 F: 18 to 24
L	9 to 21 F: 21 to 24

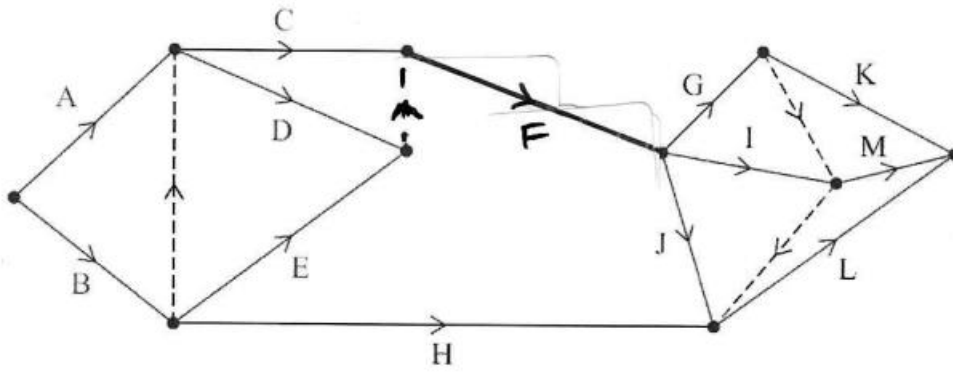
2.(a)	Prim: AE, AC, CD; BD, AF; CH, BG	M1 A1 A1 (3)
(b)	Weight of MST is 205 (minutes)	B1 (1)
(c)	J – G – B – D – C – A – E – F – H – J 28 + 32 + 28 + 27 + 29 + 28 + 36 + 39 + 42 = 289 (minutes)	M1 A1 (2)
(d)	The best upper bound is the one starting at J as 289 is less than 291	B1 (1)
(e)	The two smallest arcs incident to J are 28 and the min(x, 33) but 28 + 33 + 205 ≠ 264	B1
	205 + 28 + x = 264	M1
	x = 31	A1 (3)
		10 marks
	Notes for Question 2	
a1M1:	Prim's – first three arcs correctly chosen in order (AE, AC, CD, ...) or first four nodes {A, E, C, D, ...} correctly chosen in order. If any explicit rejections seen at some point then M1 (max) only. Order of nodes may be seen at the top of a matrix/table {1, -, 3, 4, 2, -, -, -}. Starting at any other node can score M1 only for first three arcs chosen correctly	
a1A1:	First five arcs correctly chosen in order (AE, AC, CD, BD, AF, ...) or all eight nodes {A, E, C, D, B, F, H, G} correctly chosen in order. Order of nodes may be seen at the top of a matrix so for the first two marks accept {1, 5, 3, 4, 2, 6, 8, 7} (no missing numbers)	
a2A1:	CSO – all arcs correctly stated and chosen in the correct order (with no additional arcs). They must be considering arcs for this final mark (do not accept a list of nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen)	
b1B1:	CAO for weight of MST (205) – no units required	
c1M1:	Nearest neighbour starting at J with first five nodes correct (J – G – B – D – C –) Accept arcs JG GB BD DC	
c1A1:	Correct nearest neighbour route (must return to J) (may be listed as arcs JG GB BD DC CA AE EF FH HJ) and correct length (289)	
d1B1:	Accept any wording indicating that the answer from (c) is smaller than 291 and therefore the better upper bound An indication that 289 is the minimum (of 291 and 289) – this mark is dependent on the correct value in (c) so accept an answer of the form 'the one starting at J (or the route with weight 289) as it is the least'	
e1B1:	Correct justification that the two smallest arcs incident to J are 28 and x - may calculate that the two smallest arcs must be 28 and 31 and then state that as no other arc has length 31, this must be the value of x	
e1M1:	Forming the equation: weight of MST from (b) + 28 + x = 264	
e1A1:	CAO for x	

<p>3.(a)(i)</p>		<p>M1 A1 (ABCDE) A1 (FG) A1ft (HJ)</p>
	<p>Shortest path from A to J is ABCDFGHJ</p>	<p>A1</p>
<p>(a)(ii)</p>	<p>Length of shortest path from A to J is 83 (km)</p>	<p>A1ft (6)</p>
<p>(b)</p>	<p>$B(CD)F + HJ = 30 + 5 = 35^*$ $B(CDFG)H + F(GH)J = 61 + 36 = 97$ $B(CDFGH)J + F(G)H = 66 + 31 = 97$</p>	<p>M1 A1 A1</p>
	<p>Repeat arcs: BC, CD, DF and HJ</p>	<p>A1</p>
	<p>Route length is $458 + 35 = 493$ (km)</p>	<p>A1ft (5)</p>
<p>(c)</p>	<p>The shortest path between any of the pairs of the four odd nodes (A, B, F and H) is AB (17)</p>	<p>M1</p>
	<p>So start at F and finish at H (or vice-versa)</p>	<p>A1</p>
	<p>Length of route is $458 + 17 = 475$ (km)</p>	<p>B1 (3)</p>
		<p>14 marks</p>
	<p>Special Case for Part B – using A B F H – Mark as Misread</p>	
<p>(b)</p>	<p>$AB + F(G)H = 17 + 31 = 48^*$ $A(BCD)F + B(CDFG)H = 47 + 61 = 108$ $A(BCDFG)H + B(CD)F = 78 + 30 = 108$</p>	<p>M1 A1 A1</p>
	<p>Repeat arcs: AB, FG and GH</p>	<p>A1</p>
	<p>Route length is $458 + 48 = 506$ (km)</p>	<p>A1ft (5)</p>
	<p>Remove final two A marks earned in this section so max 3/5</p>	

Notes for Question 3		
	<p>In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at D the working values must be 39 37 35 in that order (so 39 35 37 is incorrect)</p> <p>It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, ... will be penalised once (see notes below) but 1, 2, 3, 5, 6, ... is fine. Errors in the final values and working values are penalised before errors in the order of labelling</p>	
a1M1	A larger value replaced by a smaller value in the working values of at least two of the nodes C, D, F, G, H, J	
a1A1	All values at A, B, C, D and E correct and the working values in the correct order	
a2A1	All values at F and G correct and the working values in the correct order	
a3A1ft	All values in H and J correct on the follow through and the working values in the correct order. To follow through H check that the working values at H follow from the candidate's final values for the nodes that are directly attached to H (which are E and G). For example, if correct then the order of labelling of nodes E and G are 5 and 7 respectively so the working values at H should come from E and G in that order. The first working value at H should be their 45 (the Final value at E) + 47 (the weight of the arc EH), the second working value at H should be their 67 (the Final value at G) + 11 (the weight of the arc GH). Repeat the process for J (which will have working values from D, F, G and H with the order of these nodes determined by the candidate's order of labelling at D, F, G and H)	
a4A1	CAO (ABCDFGHJ)	
a5A1ft	Follow through their final value at J only – if answer is 83 but this is not the Final Value at J then A0	
b1M1	Three distinct pairings of the nodes B, F, H and J with one row correct (including total)	
b1A1	Any two rows correct including pairings and totals	
b2A1	All three rows correct including pairings and totals	
b3A1	CAO - correct arcs clearly stated and not just in their working as BC, CD, DF and HJ – must be these arcs. Do not accept BF or B(CD)F or BF via C and D	
b4A1ft	Correct route length (493) or follow through their least repeat + 458	
c1M1	Indicates the need to find the shortest path between any pair of the correct four odd nodes (A, B, F, H)	
c1A1	CAO (F, H)	
c1B1	CAO (475)	

Special Case for Part B – using A B F H – Mark as Misread		
b1M1	Three distinct pairings of the nodes A, B, F and H with one row correct (including total)	
b1A1	Any two rows correct including pairings and totals	
b2A1	All three rows correct including pairings and totals	
b3A1	CAO - correct arcs clearly stated and not just in their working as AB, FG and GH – must be these arcs. Do not accept FH or F(G)H or FH via G	
b4A1ft	Correct route length (506) or follow through their least repeat + 458	
	Remove final two A marks earned in this section so max 3/5	

4.(a)		M1 A1 A1 A1 A1 (5)
	oe (see below)	
(b)(i)	Activities A, C, D and H cannot be critical	B1
(b)(ii)	Activities B and F must be critical	B1 (2)
7 marks		
Notes for Question 4		
	Condone lack of, or incorrect, numbered events throughout. ‘Dealt with correctly’ means that the activity starts from the correct event but need not necessarily finish at the correct event, e.g. ‘L dealt with correctly’ requires the correct precedences for this activity, i.e. H and J labelled correctly and leading into the same node and L starting from that node but do not consider the end event for L. Activity on node is M0	
	If an arc is not labelled, for example, if the arc for activity E is not labelled (but the arc is present) then this will lose the first A mark and the final (CSO) A mark – they can still earn the second A mark on the bod. If two or more arcs are not labelled then mark according to the scheme. Assume that a solid line is an activity which has not been labelled rather than a dummy (even if in the correct place for where a dummy should be). Ignore incorrect or lack of arrows on the activities for the first four marks only	
a1M1	Nine activities (labelled on arc), one start and at least two dummies placed	
a1A1	Activities A, B, 1 st dummy (including correct arrow), C, D, E and H dealt with correctly	
a2A1	2 nd dummy (including correct arrow), activities F, G, I and J dealt with correctly	
a3A1	Activities K, L and M dealt with correctly (so third and fourth dummies required + arrows)	
4A1	CSO – all arrows correctly placed for each activity with one finish and at most four dummies.	
Please check all arcs carefully for arrows – if there are no arrows on any dummies then M1 only. Note that additional (but unnecessary) ‘correct’ dummies that still maintain precedence for the network should only be penalised with the final A mark if earned		
b1B1	CAO (A, C, D and H and no others)	
b2B1	CAO (B and F and no others – accept though if E mentioned)	



Alternative equivalent graph
 Activities C and D may be interchanged on either version

5.	Objective function is maximised at $C(9, 22)$ and minimised at $A(6, 8)$	B1
	Let $P = \lambda(4x + 4.5y)$	M1
	$540 = \lambda(4(9) + 4.5(22)) \Rightarrow \lambda = \dots$	M1
	$P_{\min} = \lambda(4(6) + 4.5(8))$	dM1
	$P_{\min} = 240$	A1 (5)
		5 marks
	Notes for Question 5	
1B1	Recognises that the objective function is maximised at C and minimised at A (possibly implied by later working). Award if correct coordinates $(9, 22)$ and $(6, 8)$ used in their calculations. Sight of $9a + 22b = 540$ and $6a + 8b = k$ earns this mark.	
1M1	Setting up an objective function of the form $\lambda(4x + 4.5y)$ or $\lambda(4.5x + 4y)$ Allow consideration of $4x + 4.5y$ or any multiple Considers the gradient of the objective function $-\frac{8}{9}$ Considers ratio between a and b (e.g. $4b = 4.5a$)	
2M1	A correct approach to find the objective function. ($P =$) $16x + 18y$ earns this mark (if no incorrect working seen). This may be implied by consideration of e.g. $540/135$ Candidates may adopt alternative algebraic approaches using $9a + 22b$ and forming simultaneous equations. Any algebraic approach leading to the correct answer with no incorrect working is acceptable. e.g. Setting up an equation in their λ using correct point C and their expression for P and solving for their λ (if correct $\lambda = 4$)	
3M1	Dependent on previous M mark – using point A and their objective function (of the form $\lambda(4x + 4.5y)$ where $\lambda \neq 1$) to get a value for P_{\min}	
1A1	CAO ($k = 240$)	

6.(a)	Bin 3 has the largest sum of the four bins with $72 \Rightarrow n \geq 72$	B1																																																																																											
	After the first three numbers were placed in Bin 1 (with a total of 64) the next smallest value (the 11) could not fit in Bin 1 $\Rightarrow n < 64 + 11$	B1																																																																																											
	$72 \leq n \leq 74$ oe e.g. $n = 72, 73$ or 74	B1 (3)																																																																																											
(b)	<p>middle right</p> <table style="width:100%; border-collapse: collapse;"> <tr> <td style="text-align: right; padding-right: 10px;">28</td> <td style="text-align: right; padding-right: 10px;">31</td> <td style="text-align: right; padding-right: 10px;">5</td> <td style="text-align: right; padding-right: 10px;">25</td> <td style="text-align: right; padding-right: 10px;">16</td> <td style="text-align: right; padding-right: 10px;">35</td> <td style="text-align: right; padding-right: 10px;">18</td> <td style="text-align: right; padding-right: 10px;">22</td> <td style="text-align: right; padding-right: 10px;">11</td> <td style="text-align: right; padding-right: 10px;">27</td> <td style="text-align: right; padding-right: 10px;">15</td> <td style="text-align: right; padding-right: 10px;">13</td> <td style="text-align: right;">Pivots</td> </tr> <tr> <td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td style="text-align: right;">18</td> </tr> <tr> <td style="text-align: right; 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(c)	After the 27 has been placed in Bin 2 (giving a total of 55) the 18 was placed in Bin 3 not Bin 2 indicating that $n < 55 + 18$	B1																																																																																											
	$n = 72$	dB1 (2)																																																																																											
		9 marks																																																																																											
Notes for Question 6																																																																																													
a1B1	Correct reasoning why n is at least 72 States or shows a bin (3) has total 72 and concludes that this is the least value																																																																																												
a2B1	Correct reasoning regarding the upper bound for n – that is as the second smallest value (the 11) was not put in Bin 1 then $n <$ the sum of Bin 1 + 11																																																																																												
a3B1	$n = 72, 73$ or 74																																																																																												
b1M1	Quick sort – pivots, p , selected and first pass gives $\langle p, p, \rangle p$. If only choosing 1 pivot per iteration M1 only. If sorting into descending order then mark as a misread																																																																																												
b1A1	First pass correct and next pivots chosen correctly/consistently for second pass																																																																																												
b2A1ft	Second and third passes correct (ft from their first pass and choice of pivots)																																																																																												
b3A1	CSO (including a fifth pass)																																																																																												
c1B1	Correct reasoning regarding the placement of the 18 in Bin 3 rather than Bin 2 (must explicitly mention 18)																																																																																												
c2B1	Dependent on previous B mark - CAO ($n = 72$)																																																																																												

	MISREAD – sorting into descending order													
6.(b)	middle right													
													Pivots	
	28	31	5	25	16	35	18	22	11	27	15	13	18	M1
	28	31	25	35	22	27	<u>18</u>	5	16	11	15	13	11, 35	A1
	<u>35</u>	28	31	25	22	27	<u>18</u>	16	15	13	<u>11</u>	5	15, 25	
	<u>35</u>	28	31	27	<u>25</u>	22	<u>18</u>	16	<u>15</u>	13	<u>11</u>	<u>5</u>	31	A1ft
<u>35</u>	<u>31</u>	28	27	<u>25</u>	<u>22</u>	<u>18</u>	<u>16</u>	<u>15</u>	<u>13</u>	<u>11</u>	<u>5</u>	27	A1	
<u>35</u>	<u>31</u>	28	27	<u>25</u>	<u>22</u>	<u>18</u>	<u>16</u>	<u>15</u>	<u>13</u>	<u>11</u>	<u>5</u>		(4)	
	middle left													
													Pivots	
28	31	5	25	16	35	18	22	11	27	15	13	35		
<u>35</u>	28	31	5	25	16	18	22	11	27	15	13	18		
<u>35</u>	28	31	25	22	27	<u>18</u>	5	16	11	15	13	11, 25		
<u>35</u>	28	31	27	<u>25</u>	22	<u>18</u>	16	15	13	<u>11</u>	5	15, 31		
<u>35</u>	<u>31</u>	28	27	<u>25</u>	<u>22</u>	<u>18</u>	16	<u>15</u>	13	<u>11</u>	<u>5</u>	28		
<u>35</u>	<u>31</u>	<u>28</u>	27	<u>25</u>	<u>22</u>	<u>18</u>	<u>16</u>	<u>15</u>	<u>13</u>	<u>11</u>	<u>5</u>			
Notes for Question 6														
b1M1	Quick sort – pivots, p, selected and first pass gives <p, p, >p. If only choosing 1 pivot per iteration M1 only. If sorting into descending order then mark as a misread													
b1A1	First pass correct and next pivots chosen correctly/consistently for second pass													
b2A1ft	Second and third passes correct (ft from their first pass and choice of pivots)													
b3A1	CSO (including a fifth pass)													
REMOVE final two A marks earned in this section so max 2/4														

7.(a)	$(P=) 160x + 75y + 125z$ and maximise	B1
	Subject to:	
	$x + y + z \leq 100$	B1
	$\frac{1}{4}(x + y + z) \leq x (\Rightarrow 3x - y - z \geq 0)$	M1
	$3z \leq 5y$	M1
	$2x + 1.5y + 0.75z \leq 138 (\Rightarrow 8x + 6y + 3z \leq 552) (120x + 90y + 45z \leq 8280)$	M1
	$3x - y - z \geq 0$ $3z \leq 5y$ $8x + 6y + 3z \leq 552$ $x \geq 0 \quad y \geq 0 \quad z \geq 0$	A1 (6)
(b)	Substitute $x + y + z = 100$ into $P = 160x + 75y + 125z$ and simplify	M1
	$P = 5(7x - 10y) + 12500$ so maximising a (positive) multiple of $7x - 10y$ is equivalent to minimising the negative of this expression, that is, $-(7x - 10y) = -7x + 10y^*$	A1* (2)
(c)		B1 B1 B1 B1 (4)
(d)(i)	Vertices of R are $\left(25, \frac{225}{8}\right), \left(25, \frac{127}{3}\right), (36, 24)$	M1 A1

(ii)	$\left(25, \frac{225}{8}\right)$	$\frac{-7x + 10y}{4} (106.25)$	$\frac{160x + 75y + 125z}{47875} (11,968.75)$	dM1
	$\left(25, \frac{127}{3}\right)$	$\frac{745}{3} (248.3)$	$\frac{33775}{3} (11,258.3)$	
	$(36, 24)$	-12	12560	
	So 36 acres for crop A, 24 for crop B and 40 acres for crop C			
(iii)	Maximum expected profit is (£) 12 560			A1
				(5)
				17 marks
Notes for Question 7				
	If using A, B and C throughout (a) instead of x , y and z send to review			
a1B1	CAO – expression $(160x + 75y + 125z)$ together with ‘max’ or ‘maximise’ not ‘maximum’			
a2B1	CAO $(x + y + z \leq 100)$			
a1M1	$\frac{1}{4}(x + y + z) \square x$ where \square is any inequality or equals – brackets must be present or implied by later working (accept correct equivalent unsimplified forms)			
a2M1	$3z \square 5y$ where \square is any inequality or equals. Also allow $5z \leq 3y$ for this mark (accept correct equivalent unsimplified forms)			
a3M1	$2x + 1.5y + 0.75z \square 138$ (oe) where \square is any inequality or equals. Time may be converted to minutes (must be all 4 values) (accept correct equivalent unsimplified forms)			
a1A1	All three constraints $(3x \geq y + z, 3z \leq 5y, 8x + 6y + 3z \leq 552)$ correct – must have integer coefficients with only one term in each variable. Condone omission of the trivial constraints $x \geq 0, y \geq 0, z \geq 0$ (Accept e.g. $24x + 18y + 9z \leq 1656$ oe if working in minutes)			
b1M1	Substitute $x + y + z = 100$ into their linear objective function and simplify to a single term in x and a single term in y only			
b1A1*	Explaining why maximising the correct objective $35x - 50y (+12500)$ is equivalent to minimising $-7x + 10y$			
c1B1	Any two lines correctly drawn ($5x + 3y = 252$ should pass within half a small square of $(18, 54)$ and $(42, 14)$) ($3x + 8y = 300$ should pass within half a small square of $(20, 30)$ and $(100, 0)$) ($x + y = 100$ should pass within half a small square of $(0, 100)$, $(50, 50)$ and $(100, 0)$) ($x = 25$ must be drawn through the middle of the small square from $(24, 0)$ to $(26, 0)$)			
c2B1	Any three lines correctly drawn			
c3B1	All four lines correctly drawn (penalise any poorly drawn lines with the loss of this mark)			
c4B1	Correct R labelled – dependent on all three previous B marks			
d1M1	Attempt to find the exact coordinates (must be finding at least two pairs of coordinates) of their R – dependent on at least two B marks in (c)			
d1A1	CAO - all three exact coordinates of the correct R (accept 28.125 and $42.\bar{3}$ but not 42.3 or 42.33)			
d2M1	Point testing (objective line method is M0) – testing at least two of their vertices of their R in either $-7x + 10y$ or $160x + 75y + 125z$ - dependent on previous M mark			
d2A1	All three values of P found (either exact or decimal equivalent for either objective function) and stating the correct allocation of the three crops. Allocation must be in context for crop A, B, C not $x =, y =, z =$			
d3A1	Maximum expected profit stated correctly (units not required)			

