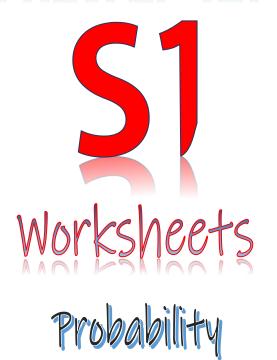
# Mathematics and the second sec

Edexcel IAL



Eng. Nagy Elraheb

# **Probability**

## Exercise 1:

- 1 Two coins are tossed. Find the probability of both coins showing the same outcome.
- 2 Two six-sided dice are rolled and their product, X, is recorded.
  - a Draw a sample space diagram showing all the possible outcomes of this experiment.
  - **b** Find the probability of each event:

i X = 24

ii X < 5

iii X is even.

- 3 The masses of 140 adult Bullmastiff dogs are recorded in a table. One dog is chosen at random.
  - **a** Find the probability that the dog has a mass of 54kg or more.
  - **b** Find the probability that the dog has a mass between 48 kg and 57 kg.

The probability that a Bullmastiff chosen at random has a mass under 53 kg is 0.54.

c Is it more or less likely that a Bullmastiff chosen at random has a mass under 53 kg? State one assumption that you have made in making your decision.

Mass, m (kg)	Frequency
$45 \le m < 48$	17
$48 \le m < 51$	25
$51 \le m < 54$	42
$54 \le m < 57$	33
$57 \le m < 60$	21
$60 \le m < 63$	2

Hint
Use interpolation.

- 4 The lengths, in cm, of 240 koalas are recorded in a table. One koala is chosen at random.
  - a Find the probability that the koala is female.
  - **b** Find the probability that the koala is less than 80 cm long.
  - c Find the probability that the koala is a male between 75 cm and 85 cm long.

Koalas under 72 cm long are called juvenile.

**d** Estimate the probability that a koala chosen at random is juvenile. State one assumption you have made in making your estimate.

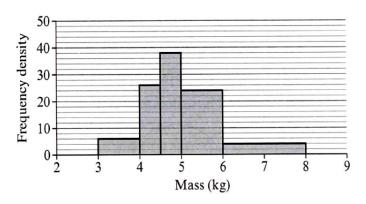
Length, l (cm)	Frequency (male)	Frequency (female)
65 ≤ <i>l</i> < 70	4	14 '
70 ≤ <i>l</i> < 75	20	15
75 ≤ <i>l</i> < 80	24	32
80 ≤ <i>l</i> < 85	47	27
85 ≤ <i>l</i> < 90	31	26

- 5 The histogram shows the distribution of masses, in kg, of 70 adult cats.
  - a Find the probability that a cat chosen at random has a mass more than 5 kg.

(2 marks)

**b** Estimate the probability that a cat chosen at random has a mass less , than 6.5 kg.

(3 marks)



### Exercise 2:

- 1 There are 25 students in a tutor group at International College. There are 16 students in the tutor group studying Arabic, 14 studying English, and 6 students studying both English and Arabic.
  - a Draw a Venn diagram to represent this information.
  - **b** Find the probability that a randomly chosen student in the tutor group:

i studies English

ii studies English and Arabic

iii studies English but not Arabic

iv does not study English or Arabic.

2 There are 125 diners in a restaurant who were surveyed to find out if they had ordered garlic bread, pasta or cheesecake:

15 had ordered all three items

20 had ordered pasta and cheesecake

43 had ordered garlic bread

26 had ordered garlic bread and cheesecake

40 had ordered pasta

25 had ordered garlic bread and pasta

- 44 had ordered cheesecake
- a Draw a Venn diagram to represent this information.
- **b** A diner is chosen at random. Find the probability that the diner ordered:

all three items

ii pasta but not cheesecake and not garlic bread

iii garlic bread and pasta but not cheesecake iv none of these items.

3 A group of 275 people at a music festival were asked if they play guitar, piano or drums:

1 person plays all three instruments

15 people play piano only

65 people play guitar and piano

20 people play guitar only

10 people play piano and drums

35 people play drums only

- 30 people play guitar and drums
- a Draw a Venn diagram to represent this information.
- **b** A festival goer is chosen at random from the group.

Find the probability that the person chosen:

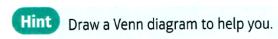
plays the piano

ii plays at least two of the instruments

iii plays exactly one of the instruments

iv plays none of the instruments.

- 4 The probability that a child in a school has blue eyes is 0.27 and the probability that the child has black hair is 0.35. The probability that the child will have black hair or blue eyes or both is 0.45. A child is chosen at random from the school. Find the probability that the child has:
  - a black hair and blue eyes
  - **b** black hair but not blue eyes
  - c neither feature.



0.35

- 5 A patient going into a doctor's waiting room reads Hiya magazine with probability 0.6 and Dakor magazine with probability 0.4. The probability that the patient reads either one or both of the magazines is 0.7. Find the probability that the patient reads:
  - a both magazines

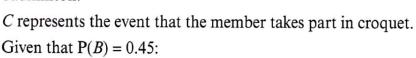
(2 marks)

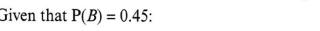
**b** Hiva magazine only.

(2 marks)

6 The Venn diagram shows the probabilities of members of a sports club taking part in various activities.

A represents the event that the member takes part in archery. B represents the event that the member takes part in badminton.





(1 mark)

0.05

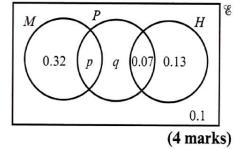
(2 marks)

- $\mathbf{a}$  find x
- **b** find v.
- 7 The Venn diagram shows the probabilities that students at a junior college study certain subjects.

M represents the event that the student studies Mathematics. P represents the event that the student studies Physics.

H represents the event that the student studies History.

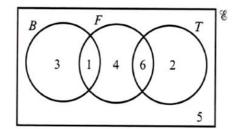
Given that P(M) = P(P), find the values of p and a.



0.1 (0.2)

# Exercise 3:

- 1 Events A and B are mutually exclusive. P(A) = 0.2 and P(B) = 0.5
  - a Draw a Venn diagram to represent these two events.
  - **b** Find P(A or B).
  - c Find P(neither A nor B).
- 2 Two fair dice are rolled and the result on each one is recorded. Show that the events 'the sum of the scores on the dice is 4' and 'both dice land on the same number' are not mutually exclusive.
- 3 P(A) = 0.5 and P(B) = 0.3. Given that events A and B are independent, find P(A and B).
- 4 P(A) = 0.15 and P(A and B) = 0.045. Given that events A and B are independent, find P(B).
- The Venn diagram shows the number of children in a play group that like playing with bricks (B), action figures (F) or trains (T).
  - a State, with a reason, which two types of toy are mutually exclusive.
  - **b** Determine whether or not the events 'plays with bricks' and 'plays with action figures' are independent.

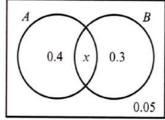


- 6 The Venn diagram shows the probabilities that a group of students like pasta (A) or pizza (B).
  - a Write down the value of x.

(1 mark)

**b** Determine whether the events 'likes pasta' and 'likes pizza' are independent.

(3 marks)



- 7 S and T are two events such that P(S) = 0.3, P(T) = 0.4 and P(S) but not T = 0.18
  - a Show that S and T are independent.
  - **b** Find:

i P(S and T)

ii P(neither S nor T).

8 W and X are two events such that P(W) = 0.5, P(W and not X) = 0.25 and P(neither W nor X) = 0.3. State, with a reason, whether W and X are independent events.

(3 marks)

9 The Venn diagram shows the probabilities of members of a social club taking part in charitable activities.

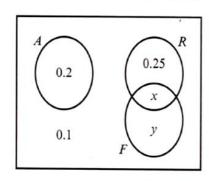
A represents taking part in an archery competition.

R represents taking part in a raffle.

F represents taking part in a fun run.

The probability that a member takes part in the archery competition or the raffle is 0.6.

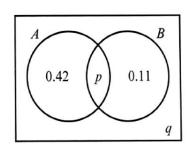
- **a** Find the value of x and the value of y.
- **b** Show that events R and F are not independent.



(2 marks)

(3 marks)

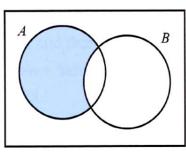
10 In the Venn diagram shown here, given that events A and B are independent, find the two possible values for p and q.



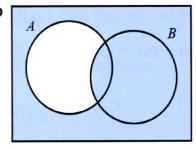
# Exercise 4:

1 Use set notation to describe the area shaded blue in each of these Venn diagrams:

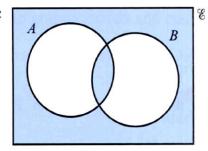
a



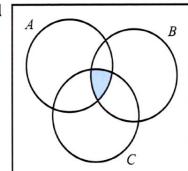
b



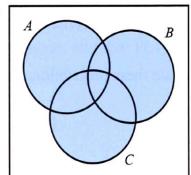
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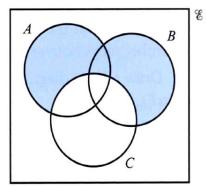
d



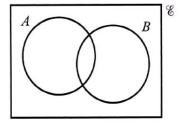
e



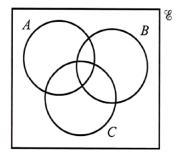
f



- 2 On copies of this Venn diagram, shade:
  - $\mathbf{a} \ A \cup B'$
  - **b**  $A' \cap B'$
  - c  $(A \cap B)'$



- 3 On copies of this Venn diagram, shade:
  - a  $(A \cap B) \cup C$
  - **b**  $(A' \cup B') \cap C$
  - c  $(A \cap B \cap C')'$



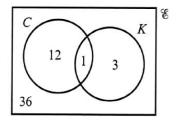
4 A card is chosen at random from a pack of 52 playing cards. C is the event 'the card chosen is a club' and K is the event 'the card chosen is a King'.

The Venn diagram shows the number of outcomes for each event.

Find:

- $\mathbf{a} \ \mathbf{P}(K)$
- **b** P(C)
- c  $P(C \cap K)$

- **d**  $P(C \cup K)$
- e P(C')
- $f P(K' \cap C)$



5 A and B are two events. P(A) = 0.5, P(B) = 0.2 and  $P(A \cap B) = 0.1$  Find:

- a  $P(A \cup B)$
- **b** P(B')

Hint

Draw a Venn diagram.

- c  $P(A \cap B')$
- **d**  $P(A \cup B')$
- 6 C and D are two events. P(D) = 0.4,  $P(C \cap D) = 0.15$  and  $P(C' \cap D') = 0.1$  Find:
  - a  $P(C' \cap D)$
- **b**  $P(C \cap D')$
- $\mathbf{c} \ \mathbf{P}(C)$
- **d**  $P(C' \cap D')$
- 7 The probability that a member of a sports club plays hockey (H) is 0.5 and the probability that the member plays cricket (C) is 0.4. The probability that the member plays both sports is 0.25
  - a Draw a Venn diagram to illustrate these probabilities.
  - b Find:
    - i  $P(H \cup C)$
- ii  $P(H' \cap C)$
- iii  $P(H \cup C')$
- 8 A bag contains 50 counters numbered from 1 to 50. The counters are either red or blue. A counter is picked at random. The two events R and E are the events 'counter is red' and 'counter is even-numbered' respectively. Given that n(R) = 17, n(E) = 30 and  $n(R \cup E) = 40$ ,
  - a draw a Venn diagram to illustrate the outcomes.
  - b Find:
    - i  $n(R \cap E)$
    - ii  $P(R' \cap E')$
    - iii  $P((R \cap E)')$

Watch out n(R) represents the **number** of outcomes in the event R, whereas P(R) represents the **probability** that the event R occurs.

**9** A, B and C are three events with P(A) = 0.55, P(B) = 0.35 and P(C) = 0.4.  $P(A \cap C) = 0.2$ . Given that A and B are mutually exclusive and B and C are independent, a draw a Venn diagram to illustrate the probabilities. (4 marks) **b** Find: i  $P(A' \cap B')$ (1 mark) (1 mark) ii  $P(A \cup (B \cap C'))$ (1 mark) iii  $P((A \cap C)' \cup B')$ 10 A, B and C are three events with P(A) = 0.25, P(B) = 0.4, Problem-solving  $P(C) = 0.45 \text{ and } P(A \cap B \cap C) = 0.1$  $\emptyset$  is the empty set.  $P(\emptyset) = 0$ . Given that A and B are independent, B and C are independent, and  $A \cap B' \cap C = \emptyset$ , a draw a Venn diagram to illustrate the probabilities. (4 marks) **b** Find: i  $P(A' \cap (B' \cup C))$ (1 mark) (1 mark) ii  $P((A \cup B) \cap C)$ c State, with reasons, whether or not events A' and C are independent. (2 marks) 11 Members of a school book club read either mysteries (M), graphic novels (G) or epic fiction (E). P(M) = 0.5, P(G) = 0.4 and P(E) = 0.6. Given that no one reads both graphic novels and epic fiction and that  $P(M \cap G) = 0.3$ , a draw a Venn diagram to illustrate these probabilities. (4 marks) **b** Find: (2 marks) i  $P(M \cup G)$ ii  $P((M \cap G) \cup (M \cap E))$ c Are the events G' and M independent? You must justify your answer. (2 marks) 12 Given that events A and B are independent and that P(A) = x and P(B) = y, find, in terms of x and y: (2 marks) a  $P(A \cap B)$ **b**  $P(A \cup B)$ (2 marks) (2 marks) c  $P(A \cup B')$ 

# Exercise 5:

1 The two-way table shows the fast-food preferences of 60 students in a school.

	Pizza	Curry	Total
Year 10	11	18	29
Year 11	14	17	31
Total	25	35	60

Find:

- **a** P(Year 10)
- **b** P(Curry | Year 10)
- c P(Curry | Year 10)
- d P(Pizza | Year 11)
- 2 In a sports club, there are 75 members, of whom 32 are adults. Of the adult members, 15 play badminton and 17 play squash. There are 22 teenagers who play squash and the rest play badminton.
  - a Draw a two-way table to illustrate this situation.
  - **b** Find:
    - i P(Teenager | Squash)
- ii P(Adult | Badminton)
- iii P(Squash | Adult)
- 3 Eighty children are asked about their favourite ice-cream flavour. Of the 45 girls, 13 like vanilla, 12 like chocolate and the rest like strawberry. Of the boys, 2 like vanilla and 23 like strawberry. The rest like chocolate.
  - a Draw a two-way table to show this situation.
- **b** Find:
  - i P(Boy | Strawberry)
- ii P(Girl | Vanilla)
- iii P(Chocolate | Boy)
- 4 A red and a blue spinner each have four equally likely outcomes, numbered 1 to 4. The two spinners are spun at the same time, and the sum of the numbers shown, X, is recorded.
  - a Draw a sample space diagram for X.
  - **b** Find:
    - i P(X = 5)
- ii P(X = 3 | Red spinner is 2)
- iii P(Blue spinner is  $3 \mid X = 5$ )
- 5 Two fair six-sided dice are thrown and the product is recorded.
  - a Draw a sample-space diagram to illustrate the possible outcomes.
  - **b** Given that the first dice shows a 5, find the probability that the product is 20.
  - c Given that the product is 12, find the probability that the second dice shows a 6.
  - d Explain the importance of the word 'fair' in this context.
- 6 A card is drawn at random from a pack of 52 playing cards. Given that the card is a diamond, find the probability that the card is an Ace.

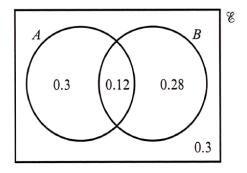
7	Two coins are flipped and the results are recorded. Given that one coin lands on a head, find the probability of:			ad,	
	a two heads b a h	ead and a tail			
	c State one modelling assumption	n used in you	r calculations.		
8	8 120 students are asked about their viewing habits. 56 say they watch sports (S) and 77 say the watch dramas (D). Of those who watch dramas, 18 also watch sports.				7 say they
	a Draw a two-way table to show	this informat	ion.		(2 marks)
	<b>b</b> One student is chosen at rando				,
	i P( <i>D</i> ′)				(1 mark)
	ii $P(S' \cap D')$				(1 mark)
	iii $P(S \mid D)$				(1 mark)
	iv $P(D' \mid S)$				(1 mark)
9	A hiking group is made up of 63 women and 47 men. 26 of the women and 18 of the men use a walking stick.				men use a
	a Draw a two-way table to show	this informati	on.		(2 marks)
	<b>b</b> One hiker is chosen at random	. Find:			,
	i P(Uses a stick)				(1 mark)
	ii P(Uses a stick   Female)				(1 mark)
	iii P(Male   Uses a stick)				(1 mark)
10	One owner is chosen at random.  A veterinary surgery has 750 registered pet owners. Of these, 450 are female. 320 of the pet owners own a cat and 250 own a budgie. Of the remaining pet owners, 25 are males who ow another type of pet. No one owns more than one type of pet. 175 female owners have a cat. One owner is chosen at random.  F is the event that an owner is female.  B is the event that an owner has a budgie.				nales who own
	C is the event that an owr	er nas a cat.			
	Find:				
	<b>a</b> $P(B' \cap C')$ <b>b</b> $P(B \mid A)$	F') c	$P(F' \mid C)$	<b>d</b> $P((B' \cap C') \mid F)$	)

# Exercise 6:

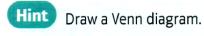
1 The Venn diagram shows the probabilities for two events, A and B.

Find:

- a  $P(A \cup B)$
- **b**  $P(A \mid B)$
- $\mathbf{c} \ \mathbf{P}(B \mid A')$
- **d**  $P(B \mid A \cup B)$



- 2 C and D are two events such that P(C) = 0.8, P(D) = 0.4 and  $P(C \cap D) = 0.25$ 
  - a Draw a Venn diagram showing the probabilities for events C and D.
  - **b** Find:
    - i  $P(C \cup D)$
- ii  $P(C \mid D)$
- iii  $P(D \mid C)$
- iv P(D' | C')
- 3 S and T are two events such that P(S) = 0.5 and P(T) = 0.7
  - a Given that S and T are independent, draw a Venn diagram showing the probabilities for events S and T.
  - **b** Find:
    - i  $P(S \cap T)$
- ii  $P(S \mid T)$
- iii P(T | S') iv  $P(S | S' \cup T')$
- 4 120 members of a youth club play either snooker (A) or pool (B) or neither. Given that 65 play snooker, 50 play pool and 20 play both, find:

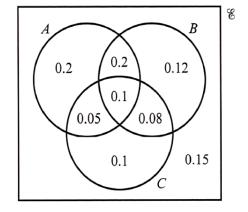


- a  $P(A \cap B')$
- **b**  $P(A \mid B)$
- $\mathbf{c} \ \mathbf{P}(B \mid A')$
- **d**  $P(A \mid A \cup B)$
- 5 The eating tastes of 80 cats are recorded. 45 like Feskers (F) and 32 like Whilix (W). 12 cats like neither. One cat is chosen at random. Find:
  - a  $P(F \cap W)$
- **b**  $P(F \mid W)$
- $\mathbf{c} \ \mathbf{P}(W|F)$
- d P(W'|F')

6 The Venn diagram shows the probabilities of three events, A, B and C.

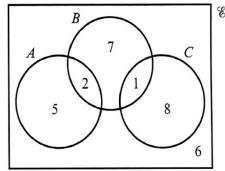
Find:

- $\mathbf{a} \ \mathbf{P}(A \mid B)$
- **b**  $P(C \mid A')$
- **c**  $P((A \cap B) \mid C')$  **d**  $P(C \mid (A' \cup B'))$



7 The Venn diagram shows the number of students in a class who watch any of 3 popular TV programmes A, B and C.

One of these students is selected at random. Given that the student watches at least one of the TV programmes, find the probability that the student watches:



- a programme C
- **b** exactly two of the programmes.
- c Determine whether or not watching programme B and watching programme C are independent.

- (2 marks)
- (2 marks)
- (3 marks)

**Problem-solving** 

If P(A|B) = P(A) then events A and B are independent.

- 8 Three events, A, B and C are such that A and B are mutually exclusive, and B and C are independent. P(A) = 0.2, P(B) = 0.6 and P(C) = 0.5. Given that  $P(A' \cap B' \cap C') = 0.1$ ,
  - a draw a Venn diagram to show the probabilities for events A, B and C.

(4 marks)

- **b** Find:
  - i  $P(A \mid C)$

(1 mark)

ii  $P(B \mid C')$ 

(1 mark)

iii  $P(C \mid (A \cup B))$ 

(1 mark)

- 9 A doctor completes a medical study of 100 people, 5 of whom are known to have an illness and 95 of whom are known not to. A diagnostic test (a test to identify illness) is applied. All 5 of the people with the illness test positive, and 10 people without the illness also test positive. Given that event A = person has the disease and event B = person tests positive,
  - a draw a Venn diagram to represent this situation.

(3 marks)

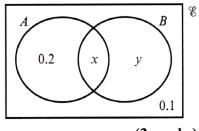
**b** Calculate  $P(A \mid B)$ 

(2 marks)

c With reference to your answer to part b, comment on the usefulness of the diagnostic test.

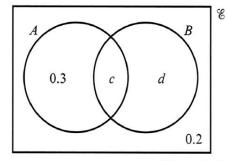
(2 marks)

- 10 Events A and B are such that P(A) = 0.6 and P(B) = 0.7. Given that  $P(A' \cap B') = 0.12$ , find:
  - a  $P(B \mid A')$
- **b**  $P(B \mid A)$
- **c** Explain what your answers to parts **a** and **b** tell you about events A and B.
- 11 The Venn diagram shows the probabilities for two events, A and B. Given that  $P(A \mid B) = P(B')$ , find the values of x and y.



(3 marks)

12 The Venn diagram shows the probabilities for events A and B. Given that  $P(A \mid B) = P(A')$ , find the values of c and d.



(3 marks)

# Exercise 7:

**a** P(A | B)

**b**  $P(A' \cap B)$ 

1	A and B are two events where $P(A) = 0.4$ , $P(B) = 0.5$ and $P(A \cup B) = 0.6$				(2) = 0.6
	Find:				
	a $P(A \cap B)$	<b>b</b> P(A')	)	c $P(A \cup B')$	<b>d</b> $P(A' \cup B)$
2	a Find $P(C \cup D)$ .			(D) = 0.65  and  P(C)	$\cap D) = 0.4$
	b Draw a Venn di $P(C' \cap D')$	<b>ii</b> P(C	D)		
	c Explain why ev	ents $C$ and $D$ a	are not indepe	endent.	
3	$E$ and $F$ are two e a Find $P(E \cup F)$ .		E) = 0.7, $P(F$	$P(E \cap F) = 0.8 \text{ and } P(E \cap F)$	") = 0.6
	<b>b</b> Draw a Venn d <b>i</b> $P(E \cup F')$	_		iii $P(E F')$	
4	There are two eve Find:	nts $T$ and $Q$ w	here $P(T) = F$	$P(Q) = 3P(T \cap Q)$ , as	$nd P(T \cup Q) = 0.75$
	a $P(T \cap Q)$	<b>b</b> P(T)	c $P(Q')$	<b>d</b> $P(T' \cap Q')$	e $P(T \cap Q')$
5	70% have a freeze	er, 20% have a c	lishwasher and	d 80% have either a c	out. The survey showed that lishwasher or a freezer or sehold in Istanbul has both
6	A and $B$ are two $A$	events such tha		P(B) = 0.5  and  P(A B) $P(A' \cap B)$	) = 0.4. Find:
7	Let A and B be ex	vents such that	$P(A) = \frac{1}{4}, P(B)$	$= \frac{1}{2} \text{ and } P(A \cup B) =$	$\frac{3}{5}$

c  $P(A' \cap B')$ 

8	C and D are two events where $P(C D) = \frac{1}{3}$ , $P(C D') = \frac{1}{5}$ and $P(D) = \frac{1}{4}$ . Find:				
	a $P(C \cap D)$	<b>b</b> $P(C \cap D')$	<b>c P</b> (C)		
	$\mathbf{d} \ \mathbf{P}(D \mid C)$	e $P(D' \mid C)$	$\mathbf{f} \ \mathbf{P}(D' \mid C')$		
9	Given that $P(A) = 0$	0.42, P(B) = 0.37  and  P(A)	$\cap$ <i>B</i> ) = 0.12, find:		
	a $P(A \cup B)$			(2 marks)	
	<b>b</b> $P(A \mid B')$			(2 marks)	
	The event $C$ has $P($	(C) = 0.3			
	The events B and C	Care mutually exclusive ar	and the events $A$ and $C$ are ind	ependent.	
	c Find $P(A \cap C)$			(2 marks)	
	d Draw a Venn dia	agram to illustrate the ever	its $A$ , $B$ and $C$ ,		
	giving the proba	bilities for each region.		(4 marks)	
	e Find $P((A' \cup C))$	<b>'</b> )		(2 marks)	
10 Three events A, B and C are such that $P(A) = 0.4$ , $P(B) = 0.7$ , $P(C)$ Given that A and C are mutually exclusive and that B and C are in					
	a $P(B \cap C)$			(1 mark)	
	<b>b</b> $P(B \mid C)$			(1 mark)	
	$\mathbf{c} \ \mathbf{P}(A \mid B')$			(1 mark)	
	$\mathbf{d} \ \mathbf{P}((B \cap C)   A')$			(1 mark)	
11	A is the event the	a are sometimes late for so at Fatima is late for school at Gayana is late for school		e defined as follows:	
	P(A) = 0.3, P(B) = 0	0.7 and $P(A' \cap B') = 0.1$ . O	n a randomly selected day, fir	nd the probability that:	
	a both Fatima and	l Gayana are late to schoo	1	(1 mark)	
	<b>b</b> Fatima is late to	school given that Gayana	is late to school.	(2 marks)	
	Their teacher suspe	ects that Fatima and Gaya	na being late for school is lin	ked in some way.	
	c Comment on his	s suspicion, showing your	working.	(2 marks)	

12 José and Cristiana play darts on the same team. The events J and C are defined as follows:

J is the event that José wins his match.

C is the event that Cristiana wins her match.

$$P(J) = 0.6$$
,  $P(C) = 0.7$  and  $P(J \cup C) = 0.8$ 

Find the probability that:

- a both José and Cristiana win their matches (1 mark)
- b José wins his match given that Cristiana loses hers (2 marks)
- c Cristiana wins her match given that José wins his. (2 marks)
- **d** Determine whether or not the events J and C are independent.

You must show all your working. (2 marks)

### Exercise 8:

- 1 Kaan takes part in two cycle races. The probability that he wins the first race is 0.6. The probability that he wins the second race is 0.7. Work out the probability that Kaan wins at least one race.
- 2 Chaiwat either walks to work or goes by bus. On any day that he goes to work, the probability that he walks is 0.6. When he walks, the probability that he is late is 0.1. When he goes by bus, the probability that he is late is 0.3.
  - a Complete a probability tree diagram.
  - **b** Calculate the probability that he goes to work by bus and is late.
  - c Calculate the probability that he is not late for work.
- 3 A bag contains 20 coins.

Ten are 25-cent coins

Six are 10-cent coins

Four are 5-cent coins

Sadia takes two of the coins at random from the bag.

Hint Draw a tree diagram.

- a Work out the probability that Sadia takes two 5-cent coins.
- **b** Work out the probability that the total value of the two coins is 20 cents or less.

- 4 A bag contains five red and four blue tokens. A token is chosen at random, the colour recorded and the token is not replaced. A second token is chosen and the colour recorded.
  - a Draw a tree diagram to illustrate this situation.

Find the probability that:

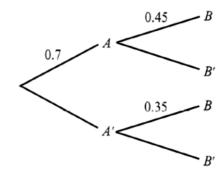
- b the second token is red, given that the first token is blue
- c the first token is red, given that the second token is blue
- d the first token is blue, given that the tokens are different colours
- e the tokens are the same colour, given that the second token is red.
- 5 A and B are two events such that P(B|A) = 0.45, P(B|A') = 0.35 and P(A) = 0.7
  - a Copy and complete the tree diagram representing this information.



i  $P(A \cap B)$ 

ii  $P(A' \cap B')$ 

iii  $P(A \mid B)$ 



- 6 A box of 24 chocolates contains 10 dark and 14 milk chocolates. Mariana chooses a chocolate at random and eats it, followed by another one.
  - a Draw a tree diagram to represent this information.

Find the probability that Mariana eats:

- b two dark chocolates
- c one dark and one milk chocolate
- d two dark chocolates, given that she eats at least one dark chocolate.
- 7 Chimamanda always goes to work by bus or by taxi. If she goes to work by bus one day, the probability she goes to work by taxi the next day is 0.4. If she goes to work by taxi one day, the probability she goes to work by bus the next day is 0.7.

Given that Chimamanda takes the bus to work on Monday, find the probability that she takes a taxi to work on Wednesday.

- 8 Aleena has two coins. One is fair, with a head on one side and a tail on the other. The second is a trick coin and has a tail on both sides. Aleena picks up one of the coins at random and flips it.
  - a Find the probability that it lands heads up.
  - b Given that it lands tails up, find the probability that she picked up the fair coin.

9	A bag contains 4 blue balls and 7 green balls. A ball is selected at random from the bag and its colour is recorded. The ball is not replaced. A second ball is selected at random and its colour is recorded.		
	a Draw a tree diagram to represent the information.	(3 marks)	
	Find the probability that:		
	b the second ball selected is green	(2 marks)	
	c both balls selected are green, given that the second ball selected is green.	(2 marks)	
10	10 In an engineering company, factories A, B and C are all producing tin sheets of the same type. Factory A produces 25% of the sheets, factory B produces 45% and the rest are produced by factory C. Factories A, B and C produce flawed sheets with probabilities 0.02, 0.07 and 0.04 respectively.		
	a Draw a tree diagram to represent this information.	(3 marks)	
	<ul> <li>b Find the probability that a randomly selected sheet is:</li> <li>i produced by factory B and flawed</li> <li>ii flawed.</li> </ul>	(2 marks) (3 marks)	
	<b>c</b> Given that a randomly selected sheet is flawed, find the probability that it was produced by factory A.	(3 marks)	
11	A genetic condition is known to be present in 4% of a population. A test is developed help find whether or not someone has the genetic condition. If a person has the cond the test is positive with probability 0.9. If a person does not have the condition, the tempositive with probability 0.02.	ition,	
	a Draw a tree diagram to represent this information.	(3 marks)	
	A person is selected at random from the population and tested for this condition.		
	<b>b</b> Find the probability that the test is negative.	(3 marks)	
	A doctor randomly selects a person from the population and tests for the condition.		
	c Given that the test is negative, find the probability that the person does have the		
	condition.	(2 marks)	
	d Comment on the effectiveness of this test.	(1 mark)	
12	On a randomly chosen day, the probabilities that Hussein travels to work by car, by b or by train are 0.1, 0.6 and 0.3 respectively. The probabilities of being late when using these methods of travel are 0.55, 0.3 and 0.05 respectively.		
	a Draw a tree diagram to represent this information.	(3 marks)	
	b Find the probability that on a randomly chosen day,		
	i Hussein travels by train and is late	(2 marks)	
	ii Hussein is late.	(2 marks)	
	c Given that Hussein is late, find the probability that he did not travel by car.	(4 marks)	

- 13 A box A contains 7 counters, of which 4 are green and 3 are blue.
  - A box B contains 5 counters, of which 2 are green and 3 are blue.
  - A counter is drawn at random from box A and placed in box B.
  - A second counter is drawn at random from box A and placed in box B.
  - A third counter is then drawn at random from the counters in box B.
  - a Draw a tree diagram to show this situation.

(4 marks)

The event C occurs when the 2 counters drawn from box A are of the same colour.

The event D occurs when the counter drawn from box B is blue.

- **b** Find P(C). (3 marks)
- c Show that  $P(D) = \frac{27}{49}$  (3 marks)
- **d** Show that  $P(C \cap D) = \frac{11}{49}$  (2 marks)
- e Hence find  $P(C \cup D)$ . (2 marks)
- f Given that all three counters drawn are the same colour, find the probability that they are all green. (3 marks)
- 14 A box of jelly beans contains 7 sweet flavours and 3 sour flavours. Two of the jelly beans are taken one after the other and eaten. Emilia wants to find the probability that both jelly beans eaten are sweet, given that at least one of them is. Her solution is shown below:

P(both jelly beans are sweet) = 
$$\frac{7}{10} \times \frac{7}{10} = \frac{49}{100}$$

P(at least one jelly bean is sweet)

= 1 - P(neither are sweet) = 1 - 
$$\frac{3}{10} \times \frac{3}{10} = \frac{91}{100}$$

P(both are sweet given at least one is sweet)

$$=\frac{\frac{49}{100}}{\frac{91}{100}}=\frac{49}{91}$$

Identify Emilia's mistake and find the correct probability.

(4 marks)