Mark Scheme (Results)

Summer 2023

Pearson Edexcel International Advanced Level
In Decision Mathematics (WDM11)
Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

The total number of marks for the paper is 75.

Edexcel Mathematics mark schemes use the following types of marks:

'M' marks
These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation, e.g. resolving in a particular direction; taking moments about a point; applying a suvat equation; applying the conservation of momentum principle; etc.

The following criteria are usually applied to the equation.

To earn the M mark, the equation
(i) should have the correct number of terms
(ii) each term needs to be dimensionally correct

For example, in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.

For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

'M' marks are sometimes dependent (DM) on previous M marks having been earned, e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

'A' marks
These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. e.g. M0 A1 is impossible.

'B' marks
These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph).

A and B marks may be f.t. – follow through – marks.

General Abbreviations

- These are some of the traditional marking abbreviations that will appear in the mark schemes:
- bod means benefit of doubt
- ft means follow through
    - the symbol $\sqrt{}$ will be used for correct ft
- cao means correct answer only
- cso means correct solution only, i.e. there must be no errors in this part of the question to obtain this mark
- isw means ignore subsequent working

- awrt means answers which round to
- SC means special case
- oe means or equivalent (and appropriate)
- dep means dependent
- indep means independent
- dp means decimal places
- sf means significant figures
- ***** means the answer is printed on the question paper
- means the second mark is dependent on gaining the first mark

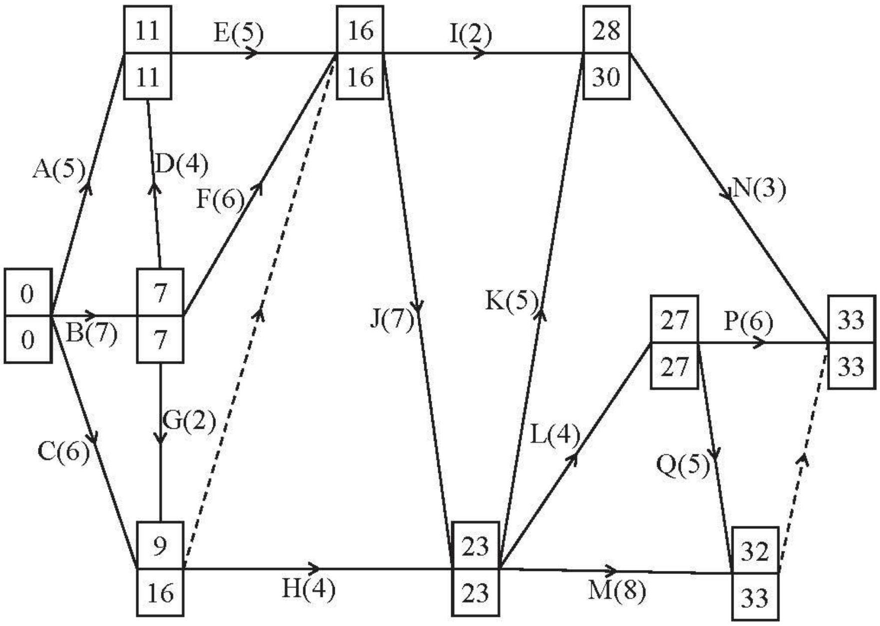
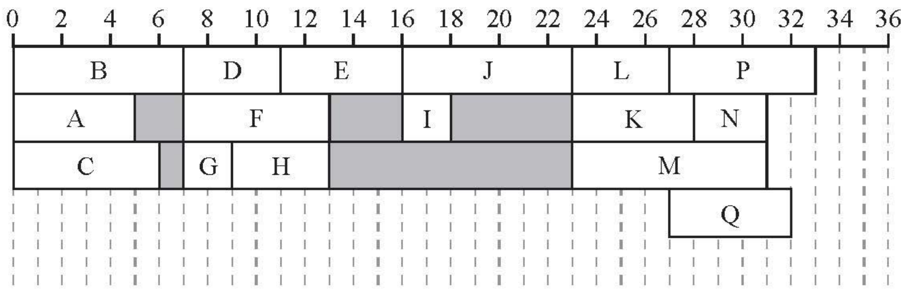
All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

If a candidate makes more than one attempt at any question:

- If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
- If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

Ignore wrong working or incorrect statements following a correct answer.

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>1. (a)</td><td>
</td><td>M1
A1
M1
A1
(4)</td></tr>
<tr><td>(b)</td><td>Activity H can be delayed by 23 – 9 – 4 = 10 (days)</td><td>B1ft
(1)</td></tr>
<tr><td>(c)</td><td>Lower bound = $\frac{5+7+6+\ldots+6+5}{33} = \frac{79}{33} = 2.393... = 3$ workers</td><td>M1 A1
(2)</td></tr>
<tr><td>(d)</td><td>e.g.

</td><td>M1 A1
A1
(3)</td></tr>
<tr><td></td><td></td><td>10 marks</td></tr>
</table>

Notes for Question 1
a1M1: All top boxes complete, values generally increasing in the direction of the arrows (so generally going from 'left to right' across the network), condone one 'rogue' value (if values do not increase in the direction of the arrows then if one value is ignored and the remaining values do increase in the direction of the arrows then this is considered to be a single rogue value). Note that all values in the top boxes could be incorrect but it can still score the M mark if the values are **increasing** in the way stated above **a1A1:** CAO – **all** values correct in the top boxes **a2M1:** All bottom boxes complete (but condone a blank box for the late event time at the end event node and/or no zero value for the late event time at the start event node for the **M** mark only). Values generally decreasing in the opposite direction of the arrows (so generally going from 'right to left' across the network), condone one 'rogue' (as described above in **a1M1**) **a2A1:** CAO – **all** values correct in the bottom boxes **b1B1ft:** Correct calculation seen for their H (provided total float is non-negative). Correct answer or the correct answer following through the event times for H with no working seen scores **B0** – must see all three numbers in their calculation (e.g. 23 – 9 – 4 = 10, or 9 + 4 = 13, 23 – 13 = 10, etc.) **c1M1:** Attempt to find lower bound: (a value in the interval [71 – 87] / their finish time) **or** (sum of all the activities / their finish time) **or** (as a minimum) an awrt 2.4 **c1A1:** CSO – requires the correct answer of 3 **and** either a **correct** calculation **or** awrt 2.4. An answer of 3 with no working scores no marks in this part **d1M1:** Not a cascade chart. 4 workers used at most, at least 10 activities placed **d1A1:** 4 workers. All 16 activities present just once. Condone at most **two** errors. An activity can give rise to at most three errors; one on duration, one on time interval and only one on IPA **d2A1:** CAO – no errors, all 16 activities present just once.

Activity	Duration	Time interval	IPA
A	5	0 – 11	-
B	7	0 – 7	-
C	6	0 – 16	-
D	4	7 – 11	B
E	5	11 – 16	A, D
F	6	7 – 16	B
G	2	7 – 16	B
H	4	9 – 23	C, G
I	2	16 – 30	C, E, F, G
J	7	16 – 23	C, E, F, G
K	5	23 – 30	H, J
L	4	23 – 27	H, J
M	8	23 – 33	H, J
N	3	28 – 33	I, K
P	6	27 – 33	L
Q	5	27 – 33	L

Question Number	Scheme	Marks
2.(a)	The 13 was used as a pivot in the first pass as all numbers greater than 13 are on the left (of the 13) and all numbers less than 13 are on the right (of the 13) (and this is not the case for any other value)	B1 **(1)**
(b)	17 33 14 25 23 28 21 13 9 6 10 33 17 25 23 28 21 14 13 9 10 **6** 33 25 23 28 21 17 14 13 10 **9** **6** 33 25 28 23 21 17 14 13 **10** **9** **6** 33 28 25 23 21 17 14 **13** **10** **9** **6** 33 28 25 23 21 17 14 13 10 9 6	M1 A1 A1 **(3)**
(c)	Bin 1: **33 28 23** Bin 2: **25 21** 17 14 6 Bin 3: 13 10 9	M1 A1 **(2)**
		6 marks

Notes for Question 2

a1B1: pivot value of 13 correctly stated + correct reasoning (give bod provided there is a clear intention of 'values greater than the pivot, pivot, values less than the pivot' e.g. 'after the first pass all the values greater than 13 are to the left and all values smaller are on the right'. Allow for **B1** a statement such as, 'values on one side are bigger than 13 and on the other are smaller', however just a statement like 'all the values greater than 13 are to the left' is **B0** – must have an indication of values on both sides of the 13)

PLEASE NOTE NO MISREADS IN PARTS (b) AND (c) – MARK ACCORDING TO THE SCHEME AND THE SPECIAL CASE IN (b)

b1M1: Bubble sort. Consistent direction, end number (6) in place and the list beginning with the correct first four numbers (33 17 25 23) after the first pass. Do check these carefully as some candidates show the result of each comparison and swap in their first pass. Consider the placement of the candidate's numbers, rather than what the candidate labels each line of their pass. For example, assume that the first time that the 6 appears at the end of the list is the end of their first pass. Their first pass must be of the form 33 17 25 23 x 6 where x is either 5, 6 or 7 numbers
b1A1: The first, second **and** third passes correct
b2A1: Fourth and fifth passes correct – **must include a fifth pass** (ISW after correctly completing the fifth pass). If after the fourth pass they state that the list is in order and simply re-write the list then **A0** (but give bod if it could be interpreted as a fifth pass)

SC in (b): Ascending sort: First two passes correct scores **M1** only in **(b)**

17 14 25 23 28 21 13 9 6 10 33 followed by 14 17 23 25 21 13 9 6 10 28 33

c1M1: The **correct** first five values placed correctly (so must be the 33 28 25 23 and the 21) and at least eight values placed in bins - condone cumulative totals for **M1** only (the bold values)
c1A1: CSO – no additional or repeated values

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>3.(a)</td><td>
<table>
<tr><th>N</th><th>A</th><th>B</th><th>C</th><th>D</th></tr>
<tr><td>4217</td><td>421.7</td><td>421</td><td>4210</td><td>7</td></tr>
<tr><td>421</td><td>42.1</td><td>42</td><td>420</td><td>1</td></tr>
<tr><td>42</td><td>4.2</td><td>4</td><td>40</td><td>2</td></tr>
<tr><td>4</td><td>0.4</td><td>0</td><td>0</td><td>4</td></tr>
<tr><td>0</td><td></td><td></td><td></td><td></td></tr>
</table>
Output values: 7, 1, 2, 4</td><td>M1
A1
A1
A1
(4)</td></tr>
<tr><td>(b)</td><td>The first value is the units digit of N, the second value is the tens digit, the third is the hundreds digit, and so on</td><td>B2, 1, 0
(2)</td></tr>
<tr><td></td><td></td><td>6 marks</td></tr>
</table>

Notes for Question 3

a1M1: At least three rows of cells completed with a correct first row – condone repeated values in all columns or a single value in each row
a1A1: CAO – the values in the second and third row correct
a2A1: CAO – fourth row correct and a zero only in the fifth *N* row
a3A1: Correct outputs (7, 1, 2, 4) – dependent on the first **four** rows being correct – the output must either be stated on the given answer line or ‘output 7, 1, 2, 4’ must be clearly written somewhere near the table (do not bod column D being circled, etc.). Condone the output being stated as 7124

b1B1: Indication that the outputs are the digits of *N*
b2B1: Indication that the digits are in the reverse order

Examples of **B1 B1**:
- The output is *N* (**or** the original input **or** 4217) in reverse order
- The output is *N* backwards
- The output is (the digits of) *N* written right to left
- The first value is the unit digit of *N*, the second value is the tens digit, and so on

Examples for **B1 B0**:
- The output is each number of *N*
- The output is *N*
- Output values are the values that make up the original input

Examples for **B0 B0**:
- The output comes from/is derived from the original input
- The algorithm removes the last digit step by step
- The output values are the last digits of *N*
- The output is the last digit of *N*
- The output is *N* + contradictory statement (e.g. ‘output is *N* or a number that is smaller than *N*’)

Example for **B0 B1** (not common):
- The output is in reverse/right to left/backwards (so no mention of *N* **or** original input **or** 4217)

Question Number	Scheme	Marks
4.(a)	$y \leqslant 2x$ and $3x + y \leqslant 30$	B1
	Correct method for finding the third boundary of the feasible region	M1
	$5y \geqslant 6x + 10$	A1 **(3)**
(b)	(6, 12)	B1
	Solving simultaneous equations to find the other two vertices	M1
	$\left(\frac{5}{2}, 5\right)$ and $\left(\frac{20}{3}, 10\right)$	A1 **(3)**
(c)	$\left(\frac{5}{2}, 5\right) \to P = 20$ $\left(\frac{20}{3}, 10\right) \to P = \frac{130}{3}$ $(6, 12) \to P = 48$	M1 A1
	Optimal vertex is (6,12) with P = 48	A1 **(3)**
(d)	$12 + 12k \,\square\, 2\left(\frac{20}{3}\right) + 10k$ where $\square$ is any inequality or equals	M1
	$k \geqslant \frac{2}{3}$	A1 **(2)**
		11 marks

Notes for Question 4

a1B1: Both correct inequalities: $y \leqslant 2x$ and $3x + y \leqslant 30$ (or equivalent if they have been rearranged)

a1M1: Correct equation for the third boundary of the feasible region, e.g. $\frac{y-20}{14-20} = \frac{x-15}{10-15}$ would score **M1** (so does not need to be simplified). Condone any inequality sign used (provided that if the inequality sign was replaced with an 'equals' then the equation ($5y = 6x + 10$) would be correct). ISW once the correct equation has been seen (so condone the correct unsimplified equation if not simplified correctly)

a1A1: CAO for third inequality – must be three terms only but need not be further simplified (e.g. $12x - 10y + 20 \leqslant 0$, $y \geqslant 1.2x + 2$ etc. scores **A1**)

b1B1: (6, 12) **or** x = 6, y = 12

b1M1: Correct method for solving their simultaneous equations (so $y = 2x$ with their $5y = 6x + 10$ **or** $3x + y = 30$ with their $5y = 6x + 10$) to find both the x and y-coordinates of at least one of the other two vertices (can be implied by either vertex stated correctly – condone non-exact values (to at least 3 sf) for this mark if no working seen)

b1A1: CAO (2.5, 5) **and** $\left(\frac{20}{3}, 10\right)$ (must be exact, so accept $(6\tfrac{2}{3}, 10)$, $(6.\dot{6}, 10)$, etc. but not (6.6666…, 10))

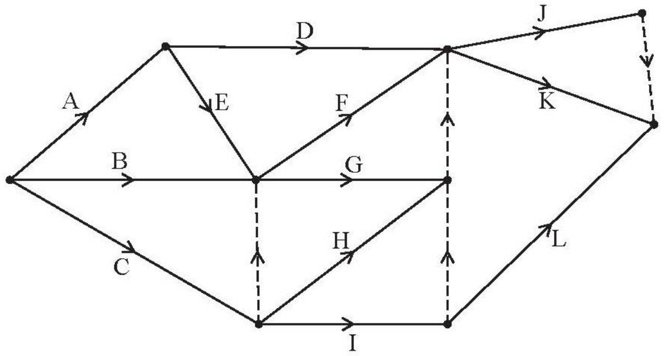
c1M1: Testing their three **vertices** (not just points in the FR) in the correct objective function ($P = 2x + 3y$). Condone one slip only when applying the objective function to one of their three vertices (so they must apply the correct objective to at least two of their three points but condone a single slip in the third)

c1A1: At least two correct (therefore exact) points tested correctly (so at least two correct values of P explicitly stated – allow awrt 43.3 when testing (20/3, 10))

c2A1: CSO all three correct (therefore exact) points tested correctly (so all three correct values of P explicitly seen – allow awrt 43.3 when testing (20/3, 10)) with a clear indication of which is the optimal vertex (this can be achieved by either making it clear that (6, 12) is the optimal point **or** that P = 48 is the maximum (as one implies the other))

d1M1: (6, 12) **and** their $\left(\frac{20}{3}, 10\right)$ (does not need to be exact) correctly substituted into $P = 2x + ky$ and compared (by comparing we mean forming an equation or any inequality). **Or** comparing $-\frac{2}{k}$ with -3 (so using the gradient of the objective line) **or** comparing $\frac{2}{k}$ with 3 **or** their reciprocals (e.g. $\frac{k}{2}$ compared with $\frac{1}{3}$ etc.) but comparing $-\frac{2}{k}$ with 3 is **M0** (by comparing we mean forming an equation or any inequality with the **correct** pairs of values)

d1A1: CAO either $k \geqslant \frac{2}{3}$ or $k > \frac{2}{3}$ **only** (or exact equivalents) – if any other answers given with $k \geqslant \frac{2}{3}$ e.g. $k \geqslant -1$ then **A0** (unless clearly rejected). Correct answer with no working can score both marks in this part. Please ensure that if working is shown that the correct answer of $k \geqslant \frac{2}{3}$ comes from correct working e.g. $-\frac{2}{k} \geqslant -3$

Question Number	Scheme	Marks
5. (a)	e.g. 	M1 A1 A1 A1 A1 **(5)**
(b)	Duration of activity K is 33 – 10 – 7 – 8 = 8 **or** the path AEFJ has a duration of < 33	M1
	Therefore the duration of J is less than 8 hours or 0 < dur(J) < 8	A1 **(2)**
		7 marks

Notes for Question 5

Condone lack of, or incorrect, numbered events throughout. ‘Dealt with correctly’ means that the activity starts from the correct event but need not necessarily finish at the correct event and **appears only once in the network**, e.g. ‘G dealt with correctly’ requires the correct precedences for this activity, i.e. B, C and E labelled correctly and leading into the same node and G starting from that node but do not consider the end event for G **so use the table below for checking as there a number of acceptable answers. Activity on node is M0**

If an arc is not labelled, for example, if the arc for activity E is not labelled (but the arc is present) then this will lose the first A mark and the final (CSO) A mark – they can still earn the second A mark on the bod. If two or more arcs are not labelled then mark according to the scheme. Assume that a solid line is an activity which has not been labelled rather than a dummy (even if in the correct place for where a dummy should be)

Ignore incorrect or lack of arrows on the activities for the first four marks only (but assume that they are in the ‘correct’ direction for checking purposes)

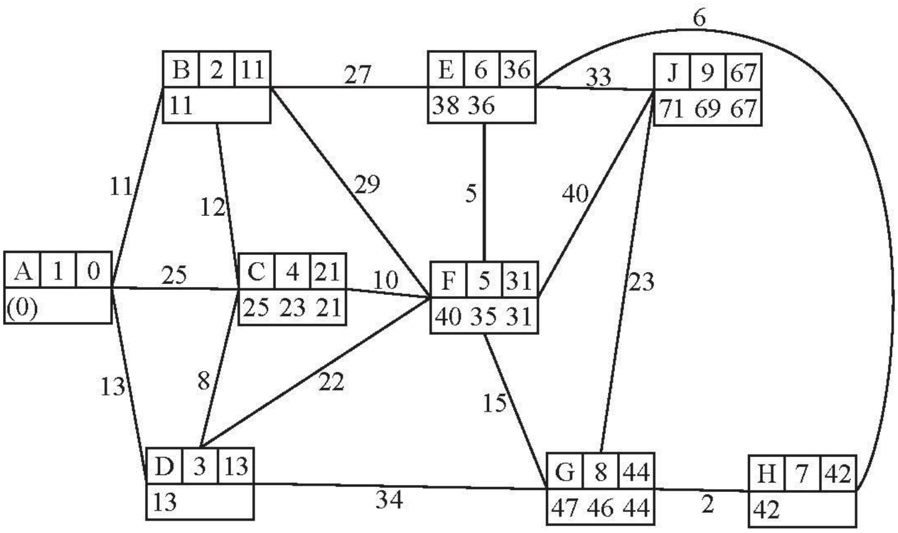
a1M1: At least seven activities (labelled on arc), one start and at least two dummies placed
a1A1: Activities A, B, C, D and E dealt with correctly
a2A1: Activities F, G, H and I dealt with correctly (so a dummy is required at the end of C + correct arrow)
a3A1: Activities J, K and L dealt with correctly (so **at least** two further dummies required + correct arrows)
a4A1: CSO – all arrows correctly placed for each activity with one finish and at most four dummies.
Please check all arcs carefully for arrows – if there are no arrows on any dummies then M1 only.
Note that additional (but unnecessary) ‘correct’ dummies that still maintain precedence for the network should only be penalised with the final A mark if earned

Extremely useful for checking (a)

Activity	A	B	C	D	E	F	G	H	I	J	K	L
IPA	-	-	-	A	A	B, C, E	B, C, E	C	C	D, F, G, H, I	D, F, G, H, I	I

b1M1: Either the correct method for calculating the duration of activity K seen (e.g. 33 – 10 – 7 – 8 or an answer of 8 can imply this mark) **or** stating the path AEFJ will have a duration < 33 or $\leqslant 33$

b1A1: Correct indication that the duration of K being 8 implies that J’s duration is < 8 (allow $\leqslant 8$ but **A0** for $\leqslant 7$). As a minimum for both marks candidates must either say that the duration of K is 8 (e.g. K = 8) **or** that the path AEFJ has a duration < 33 or $\leqslant 33$, **together** with the duration of J being either < 8 or $\leqslant 8$. A lower limit is not required but if stated then it must be either > 0 or $\geqslant 0$ **only**

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>6.(a)(i)</td><td>

Shortest path from A to J: ADCFEHGJ</td><td>M1
A1 (ABDC)
A1 (FE)
A1ft (HGJ)
A1</td></tr>
<tr><td>(a)(ii)</td><td>Length of shortest path from A to J: 67 (miles)</td><td>A1ft
(6)</td></tr>
<tr><td>(b)</td><td>AC + EJ = A(D)C + E(HG)J = 21 + 31 = 52</td><td>M1 A1</td></tr>
<tr><td></td><td>AE + CJ = A(DCF)E + C(FEHG)J = 36 + 46 = 82</td><td></td></tr>
<tr><td></td><td>AJ + CE = A(DCFEHG)J + C(F)E = 67 + 15 = 82</td><td>A1</td></tr>
<tr><td></td><td>Route length is 315 + 52 = 367 (miles)</td><td>A1ft
(4)</td></tr>
<tr><td>(c)</td><td>Pass through G a total of 3 times</td><td>B1
(1)</td></tr>
<tr><td>(d)</td><td>Difference in inspection routes is 67 – 52 = 15 (miles)</td><td>B1ft
(1)</td></tr>
<tr><td>(e)</td><td>Arcs CF and EF do not need to be repeated</td><td>B1
(1)</td></tr>
<tr><td></td><td></td><td>13 marks</td></tr>
<tr><td></td><td>Notes for Question 6</td><td></td></tr>
<tr><td colspan="3">In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at F the working values must be 40 35 31 in that order (so 40 31 35 is incorrect)
It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling</td></tr>
</table>

a1M1: A larger value replaced by a smaller value at least twice in the working values at either C, E, F, G or J
a1A1: All values at A, B, D and C correct and the working values in the correct order
a2A1: All values at F and E correct and the working values in the correct order
a3A1ft: All values in H, G and J correct on the follow through and the working values in the correct order.
To follow through G say check that the working value(s) at G follow from the candidate's final values for the nodes that are directly attached to G (which are D, F, H and J). For example, **if** correct then the order of labelling of nodes D, F and H are 3, 5 and 7 respectively so the working values at G should come from D, F and H in that order. The first working value at G should be their 23 (the Final value at D) + 34 (the weight of the arc DG), the second working value at G should be their 31 (the Final value at F) + 15 (the weight of the arc FG) and the third working value at G should be their 42 (the Final value at H) + 2 (the weight of arc GH). Repeat this exact process for H and J for the follow through for this mark
a4A1: Correct shortest path from A to J (ADCFEHGJ) **only** – not from J to A
a5A1: Follow through their final value at J **only** (condone lack of units) – if their answer is 67 but this is not their Final Value at J then **A0**

Condone for the final two marks in (a) the 'shortest path' and 'length of shortest path' written on the wrong lines

b1M1: Three distinct pairings of nodes A, C, E and J
b1A1: Any one row correct including pairing **and** total
b2A1: All three rows correct including pairings **and** totals
b3A1ft: Correct route length (367) from the correct pairing or follow through 315 + their least total from a choice of three

c1B1: CAO (3)

d1B1ft: Correct answer of 15 either from correct or no incorrect working **or** follow through (their Final value at J from (a) – their least repeat from (b)) – **this mark is dependent on having scored both M marks in (a) and (b)**

e1B1: CAO (CF, EF only)

Question Number	Scheme	Marks
7. (a)	Prim: AH, FH, EH, FG, DG, CG, BC	M1 A1 A1 **(3)**
(b)	Initial upper bound 2(201) = 402 (km)	B1ft **(1)**
(c)	A – H – F – G – D – B – C – E – A 27 + 28 + 31 + 29 + 32 + 26 + 38 + 37 = 248	M1 A1 **(2)**
(d)	Nearest neighbour starting at E has a length of 212 + x As $x \leqslant 35 \Rightarrow$ the NN route starting at E is at most 247 (km) and therefore the NN starting at E gives the better upper bound as it is less than the one starting at A (which was 248 (km))	M1 A1 **(2)**
(e)	Lower bound is given by $(201-27)+27+x=235$	M1
	$x=34$	A1
	$235 \leqslant$ optimal length $\leqslant 246$	M1 A1 **(4)**
		12 marks

Notes for Question 7

a1M1: Prim's – first three arcs correctly chosen in order (AH, FH, EH, …) **or** first four nodes {A, H, F, E, …} correctly chosen in order. If any explicit rejections seen at any point then **M1** (max) only. Order of nodes may be seen at the top of a matrix/table {1, -, -, - , 4 , 3, -, 2} so check there too. Starting at any other node apart from A can score M1 only for first three arcs chosen correctly
a1A1: First five arcs correctly chosen in order (AH, FH, EH, FG, DG, …) **or** all eight nodes {A, H, F, E, G, D, C, B} correctly chosen in order. Order of nodes may be seen at the top of a matrix so for the first two marks accept {1, 8, 7, 6, 4, 3, 5, 2} (**no** missing numbers)
a2A1: CSO – all **arcs** correctly **stated** and chosen in the correct order (with no additional arcs). They must be considering arcs for this final mark (do not accept a list of nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen)
b1B1ft: Follow through double the stated length of their MST
c1M1: NN starting at A – must have at least A – H – F – G – D –… allow if stated in terms of arcs
c2A1: CAO on length (248) **and** route (must return to A but can be stated in terms of arcs)
d1M1: Calculating the correct length of the NN route starting at E (212 + x) **and** attempting to use the range of values for x to determining the better upper bound (implied by 247 seen or [244, 247]).
d1A1: Correct best upper bound stated (the one starting at E) together with a correct comparison of **248** (possibly implicit - if this value is not explicitly stated in (d) then 248 must have been seen in (c)) with 247 (or an indication of 'at most' 247). **For those who obtained an answer of 211 in (c) and say that 212 + x is always bigger (without using the given interval for x to find the UB) then no marks in this part**
e1M1: Correct method for calculating x (which is the weight of MST from (a)/(b) – 27 + two smallest arcs incident to A (the 27 and x) equal to 235). If using the doubled value from (b) then **M0**. If not using the weight of the MST from (a) then they must be using either 174 **or** 26 + 30 + 29 + 31 + 28 + 30 **or** explicitly using the correct six arcs only (BC, CG, GD, GF, FH, HE) so not just circled in one of the tables. The correct value of x (with either no working or no incorrect working) clearly stated can imply this mark
e1A1: CAO for x (34) – **as a minimum** must have seen the calculation $201+x=235$ to award this mark
e2M1: Any indication of an interval from 235 to either 246, 247 or 248 (this mark is **not** dependent on the previous M mark)

e2A1: CAO (condone $235 < \text{optimal length} \leqslant 246$ and allow equivalent interval notation e.g. (235, 246] or [235, 246]) – this mark is dependent on all previous marks in (e) (so must have found that *x* equals 34).
The correct interval (with no others) with no supporting working scores **M0A0M1A0**
The minimum requirement for full marks is: $201 + x = 235 \Rightarrow x = 34 \therefore [235, 246]$
For those who simply state *x* = 34 (only) followed by the correct interval they score **M1A0M1A0**

Question Number	Scheme	Marks
8.(a)	(Maximise) $x+y+z$	B1
	Subject to:	
	$14x+8y+12z \leqslant 976$	B1
	$5z \geqslant 2x$	B1
	$\frac{1}{2}(x+y+z) \leqslant x (\Rightarrow -x+y+z \leqslant 0)$	M1
	$\frac{1}{5}(x+y+z) = y (\Rightarrow x-4y+z=0)$	B1
	Substituting $z=4y-x$ into objective and constraints	M1
	Maximise $(P=)5y$ Subject to: $x+28y \leqslant 488$ $7x-20y \leqslant 0$ $2x-5y \geqslant 0$ $x-4y \leqslant 0$ $x \geqslant 0 \quad y \geqslant 0$	A1 A1 **(8)**
(b)	Substituting $y=16$ into constraints gives $x \leqslant 40, x \leqslant \frac{320}{7}, x \geqslant 40, x \leqslant 64$	M1
	$\Rightarrow x=40$ and therefore the maximum number of leadership prizes is 24	A1 **(2)**
		10 marks

Notes for Question 8

a1B1: CAO ($x + y + z$) – can be implied by seeing $5y$ as the objective – do not penalise lack of 'maximise' here. Allow equal to any letter but not equal to a numerical value
a2B1: CAO ($14x+8y+12z \leqslant 976$ or an unsmplified inequality equivalent to $x+28y \leqslant 488$)
a3B1: CAO ($5z \geqslant 2x$ (or equivalent) or an unsimplified inequality equivalent to $7x-20y \leqslant 0$)
a1M1: Correct method - must see $\frac{1}{2}(x+y+z) \bullet x$ where $\bullet$ is any inequality or =. The bracket must be present or implied by later working – this mark is implied by the stating of a correct/incorrect inequality that would come from $\frac{1}{2}(x+y+z) \bullet x$ (possibly with $z = 4y – x$ substituted)
a4B1: CAO $\frac{1}{5}(x+y+z) = y$ (allow any equivalent unsimplified/simplified form for this mark) – simply stating that e.g. $x + z = 4y$ scores this mark
a2M1: Eliminating z using the correct equation $z=4y-x$ from all three non-trivial constraints and objective
a1A1: At least two simplified constraints correct + correct objective (in x/y only) – condone lack of 'maximise' for this mark - accept any equivalent forms for the constraints provided that coefficients are integers and only a single term in x and/or y
a2A1: CAO – must include 'maximise' or 'max' (but not 'maximum') and all 6 constraints – accept any equivalent forms for the constraints provided that coefficients are integers and only a single term in x and/or y

Condone using *a* for *x*, *s* for *y* and *l* for *z* – if any other letter used then send to review

Must be using inequalities in x and y only in (b) – if inequalities are still in terms of z then z must be eliminated using the correct equation (e.g. $z = 64 - x$). Attempting part (b) with equations (rather than inequalities) scores no marks (even if the correct answer of 24 is seen).

b1M1: Substituting $y = 16$ into at least two correct constraints from the following four: $x + 28y \leqslant 488$, $7x - 20y \leqslant 0$, $2x - 5y \geqslant 0$ and $x - 4y \leqslant 0$

b1A1: Correctly obtaining $x \leqslant 40$ from $x + 28y \leqslant 488$ **and** $x \leqslant \frac{320}{7}$ from $7x - 20y \leqslant 0$ **and** $x \geqslant 40$ from $2x - 5y \geqslant 0$ (condone lack of $x \leqslant 64$ from $x - 4y \leqslant 0$) then stating/implying that $x = 40$ and therefore $z = 24$ (or equivalent in context e.g. 24 leadership prizes)

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