

Mathematics

Edexcel IAL

S1

Worksheet Answers

Discrete Random Variables

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Discrete Random Variables

Exercise 1:

- 1 a This is not a discrete random variable, since height is a continuous quantity.
 b This is a discrete random variable, since it is always a whole number and it can vary.
 c This is not a variable at all, since the number of days in a week is always 7.

2 0, 1, 2, 3, 4

3 a (2, 2) (2, 3) (3, 2) (3, 3)

b i

x	4	5	6
$P(X = x)$	0.25	0.5	0.25

ii

$$P(X = x) = \begin{cases} 0.25, & x = 4, 6 \\ 0.5, & x = 5 \end{cases}$$

4 $\frac{1}{12}$

5 $k + 2k + 3k + 4k = 1$,

so $10k = 1$, so $k = \frac{1}{10}$.

6 a 0.125 b 0.875

7 a 0.3

b

x	-2	-1	0	1	2
$P(X = x)$	0.1	0.1	0.3	0.3	0.2

c 0.7

8 0.25

9 a 0.02 b 0.46 c 0.56

10 a 0.625 b 0.375 c 0

11 a

s	1	2	3	4
$P(S = s)$	$\frac{2}{3}$	$\frac{2}{9}$	$\frac{2}{27}$	$\frac{1}{27}$

b $\frac{1}{9}$

12 a

x	$P(X = x)$
0	0.07776
1	0.2592
2	0.3456
3	0.2304
4	0.0768
5	0.01024

b

y	$P(Y = y)$
0	0.32768
1	0.4096
2	0.2048
3	0.0512
4	0.0064
5	0.00032

c

z	$P(Z = z)$
1	0.4
2	0.24
3	0.144
4	0.0864
5	0.1296

13 a The sum of the probabilities is not 1.

b $2\frac{22}{61}$

Exercise 2:

1 a

x	1	2	3	4	5	6
$F(x)$	0.1	0.2	0.35	0.60	0.9	1

b 0.9

c 0.2

2 a

x	1	2	3	4	5	6
$P(X = x)$	0.1	0.1	0.25	0.05	0.4	0.1

b 0.5

c 0.4

3 a $k = \frac{1}{18}$

b

x	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{18}$	$\frac{1}{18}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{5}{18}$	$\frac{5}{18}$

c $\frac{7}{18}$

d $\frac{4}{9}$

e $\frac{1}{18}$

4 a $\alpha = 0.3$

b

x	-2	-1	0	1	2
$P(X = x)$	0.1	0.1	0.25	0.25	0.3

c 0.45

5 a $\frac{5}{6}$

c

x	1	2	3	4	5
$P(X = x)$	$\frac{2}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

6 a $k = 1$

b

x	1	2	3
$P(X = x)$	$\frac{4}{16}$	$\frac{5}{16}$	$\frac{7}{16}$

Exercise 3:

1 a $E(X) = 4.6, E(X^2) = 26$

b $E(X) = 0.3, E(X^2) = 2.5$

2 $E(X) = 4, E(X^2) = 18.2$

3 a

x	2	3	6
$P(X = x)$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

x	2	3	6
$P(X = x)$	$\frac{1}{4}$	$\frac{1}{9}$	$\frac{1}{36}$

b $E(X) = 3, E(X^2) = 11$

c $(E(X)^2) = 9$, therefore $(E(X)^2) \neq E(X)^2$

4 a

x	1	2	3	4	5
$P(X = x)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{16}$

b $E(X) = 1.9375, E(X^2) = 5.1875$

c $(E(X)^2) = 3.754$, therefore $(E(X)^2) \neq E(X)^2$

5 $a = 0.3, b = 0.3$

6 $a = 0.1, b = 0.4$

7

x	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{3}{20}$	$\frac{7}{20}$

8 \$2.78

Exercise 4:

1 a 1 b 2

2 a $E(X) = \frac{11}{6} = 1.83, \text{Var}(X) = \frac{17}{36} = 0.472$

b $E(X) = 0, \text{Var}(X) = 0.5$

c $E(X) = -0.5, \text{Var}(X) = 2.25$

3 $E(Y) = 4.5, \text{Var}(Y) = 5.25$

4 a

s	$P(S = s)$
2	$\frac{1}{36}$
3	$\frac{2}{36}$
4	$\frac{3}{36}$
5	$\frac{4}{36}$
6	$\frac{5}{36}$
7	$\frac{6}{36}$
8	$\frac{5}{36}$
9	$\frac{4}{36}$
10	$\frac{3}{36}$
11	$\frac{2}{36}$
12	$\frac{1}{36}$

b 7

c 5.833

d 2.415

5 a

d	0	1	2	3
$P(D = d)$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{8}$

b 1.25

c $\frac{15}{16} = 0.9375$

6 a $P(T = 1) = P(\text{head}) = 0.5$

$P(T = 2) = P(\text{tail, head}) = 0.5 \times 0.5 = 0.25$

$P(T = 3) = 1 - P(T = 1) - P(T = 2) = 0.25$

b $E(T) = 1.75, \text{Var}(T) = \frac{11}{16} = 0.688.$

7 a $E(X) = 4a + 2b$

b $a = 0.375, b = 0.25$

Exercise 5:

1 a

y	-1	1	3	5
$P(Y = y)$	0.1	0.3	0.2	0.4

b $E(Y) = 2.8$

c $E(X) = 2.9$ and $2E(X) - 3 = 5.8 - 3 = 2.8 = E(Y)$

2 a

y	-8	-1	0	1	8
$P(Y = y)$	0.1	0.1	0.2	0.4	0.2

b $E(Y) = 1.1$

3 a 8 b 4 c 2 d 18

e 8 f 3

4 a 6 b -9 c -2 d 1

e 9

5 a 4μ b $2\mu + 2$ c $2\mu - 2$ d $4\sigma^2$

e $4\sigma^2$

- 6 a 3.5
 b $Y = 200 + 100X$
 c $E(Y) = 550$
- 7 726.5 cm^3
- 8 a $E(X) = 1.25, \text{Var}(X) = 0.9375$
 b $E(Y) = \frac{1}{4} \times 1 + \frac{3}{8} \times 2 + \frac{1}{4} \times 4 + \frac{1}{8} \times 8 = 3$
 $E(Z) = 2E(X) + \frac{1}{2} = 3$
 c $\text{Var}(Z) = 4\text{Var}(X) = 3.75$

Exercise 6:

- 1 a $E(X) = 2$ b $\text{Var}(X) = 2$ c 1.414
 2 a $E(X) = 2$ b $\text{Var}(X) = 4$ c $E(X^2) = 8$
 3 $a = 0.1, b = 0.4$
 4 a $-0.3 \leq E(Y) \leq 0.4$
 b $a = 0.5, b = 0.2$

Exercise 7:

- 1 $E(X) = 3, \text{Var}(X) = 2$
 2 a 4
 b 4
 3 a Expectation = 3.5 and variance = $2\frac{11}{12}$
 b $\frac{2}{3}$
 4 a $\frac{3}{10}$
 b Expectation = 11 and Variance = 33
 5 A discrete uniform distribution is not likely to be a good a model for this distribution. The game depends on the skills of the player. The points are likely to cluster around the middle.
 6 a Discrete uniform distribution
 b $E(X) = 4.5$
 c $\text{Var}(X) = 5.25$
 d The expected winnings are less than the 5 cents stake.